Political Life Cycles*

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March 7, 2023

Abstract

We develop and test a formal model of leader political life cycle effects within a selectorate framework. The model leads to novel hypotheses about the provision of public goods, private goods, and freedoms over a leader's tenure in power. The analyses show, as hypothesized, that the total provision of benefits, as well as the provision of public goods and freedoms decrease significantly the longer a leader is in power while the proportion of rewards in the form of private goods (such as corruption opportunities) increases. Also as hypothesized, these life cycle effects, though highly significant, are smaller than the effects of winning coalition size.

Word Count: 9214

*We are very grateful for comments and feedback from audiences at the Politics departments at New York University, Yale University, Princeton University and from Helen Milner.

1 Introduction

All too often the optimism of a new government entering office is slowly replaced by disappointment as leaders appear to deliver fewer and fewer public-spirited policies, and, instead, increasingly line their own pockets. Such disappointment is not misplaced. Based on an extension of the selectorate theory, this paper offers a theory about the dynamics that induce political life cycle effects within any leader's winning coalition and evidence that tests the proposed life cycle effects based on how time in office affects the policies that leaders provide. We demonstrate that new leaders work relatively hard on behalf of both the people and their supporters and tend to focus on the provision of public goods. However, as tenure increases, leaders offer fewer rewards and shift their policy focus towards graft and corruption. While these temporal effects are small relative to differences across institutions, the evidence shows that the quality of governance declines as leader tenure increases. The analysis demonstrates that people are not falsely optimistic about what new leaders will deliver. Rather, as new leaders become established, they simply deliver less.

Broadly speaking, all survival-oriented political leaders allocate a portion of their government's revenue across two policy dimensions: public goods and private goods. The amounts allocated are expected to reflect the quantity required to secure the acquiescence of their subjects and the support of their coalition of essential supporters. The provision of public goods – including spending or the promotion of such policies as national security, access to education, healthcare, protection of freedoms, and other public services – is intended to sustain enough citizen support to forestall the leadership's deposition. As well, all leaders, whether they head a democracy, a monarchy, theocracy, military regime, or some other form of autocracy, must allocate some portion of revenue in the form of private goods intended to secure the loyalty of the government's essential backers. The puzzle we examine relates to how that survival-motivated allocation of public and private benefits changes the longer an

incumbent leader has been in power.

How much of a government's revenue is spent on public goods and private goods versus falling into the discretionary pot of the incumbent is known to vary greatly across regime types. Numerous studies have established, for instance, that more democratic governments spend proportionately more on public goods-oriented policies while less democratic regimes focus more spending on private goods aimed at their key backers (Lake and Baum, 2001; Deacon and Saha, 2006; Deacon, 2009). Empirical tests of public and private goods provision strongly support the theoretical contention that more democratic, more accountable regimes – that is, those that depend on a larger winning coalition – indeed do provide more public goods and fewer private goods than do their smaller-coalition counterparts. However, the predictions from these theoretical accounts of resource allocations and policy choices remain incomplete.

We extend selectorate-based explanations of governance by demonstrating theoretically and empirically that a political life cycle exists that alters the equilibrium mix of public and private goods. This life cycle also affects the total amount spent to sustain coalition loyalty and, therefore, the amount left over for the incumbent's discretionary use. In the analysis to follow, we demonstrate that, conditional on coalition size, the longer a leader has been in power, the less she spends to maintain coalition loyalty, the fewer public goods she provides, and that she allocates more to private goods relative to public goods. While we anticipate that the effect of a leader's time in office is secondary to the policy allocation incentives induced by coalition size, we also expect that the previously unstudied impact of a leader's life cycle in power adds an important, significant piece to the puzzle of how governments allocate resources. That is to say, while democratic leaders deliver more public goods than autocrats, both democrats and autocrats reduce their provision of public goods as their tenure increases. Additionally, we anticipate that the life cycle effects on policy provision materially alter the risks leaders face from deposition, coup d'état or mass uprising, but we

leave such issues for subsequent research.

2 Previous Studies of Leader-Tenure Effects

We are not, of course, the first to consider how leadership tenure shapes important domestic political phenomena whether within or outside of a selectorate perspective. We complement existing selectorate analysis that explains how a leader's fate may be tied to factors like health (Bueno de Mesquita and Smith, 2018) or threats of rebellion or coup d'état (Bueno de Mesquita and Smith, 2017). This paper contributes by considering how leaders best identify a winning coalition over time so as to provide the optimal mix of benefits to enhance survival (Riker, 1962).

Our study specifically contributes to research into how leaders' personal characteristics, especially their time in office, affect their policy choices and their political survival (Horowitz, Stam and Ellis, 2015). While our analysis is at the country level, we explicitly focus on leader behavior and the nature of individual incumbents' policy provisions (cf. McGillivray and Smith, 2008; Chiozza and Goemans, 2011). Oftentimes, leadership change also alters the relationships between nations, suggesting the importance of leaders as units of analysis (McGillivray and Stam, 2004; Mattes, Leeds and Carroll, 2015). Gelpi and Grieco (2001) also appeal to the notion of tenure or experience in making the argument that experienced leaders are less likely to be the targets of international conflict.

In a similar fashion, scholars of international relations have rigorously examined how domestic incentives shape the timing and provision of foreign policies. In the conflict literature, there has been much consideration of how the timing of conflict interacts with the electoral calendar (e.g., Gaubatz, 1991; Williams, 2013; Zeigler, Pierskalla and Mazumder, 2014; Chiozza, 2017; Smith and Spaniel, 2019). Studies on the "diversionary war hypothesis" (Levy, 1989) probe how domestic electoral incentives tempt leaders to initiate adventurous

foreign policies (Smith, 1996; Tarar, 2006). In addition, others investigate how political business cycles and macroeconomic policies are manipulated for electoral gain (e.g., Nordhaus, 1975; Alesina, 1987; Rogoff, 1990; Schultz, 1995). These studies make clear that leader tenure plays an important role in understanding key political phenomena.

Our work is distinct from also but connects to the political economy literature on term limits. Many democracies and even a few autocracies have introduced term limits over the past several decades. The lifecycle results that we present here suggest a possible reason for such term limits. Since leaders reduce their provision of public goods the longer they are in office and public goods are an especially important component of benefits in democracies, term limits may be a way to restrict the loss of public goods provision by forcing turnover in leaders, thereby ensuring an increase in public goods with each new incumbent (Ashworth, 2012).

With the insights of previous studies into leader tenure in mind, we focus attention on how and why the provision of public and private goods changes the longer an incumbent remains in power. We turn now to modeling how a leader's life cycle in power alters policy allocations.

3 A Political Life Cycle Model

In developing a model of life cycle effects on resource allocations we begin with the formal foundations of the selectorate theory (Bueno de Mesquita et al., 2003). We assume some familiarity with selectorate theory and so provide here only a minimal summary of its essential features. Critically, leaders are assumed to have their eye on survival in office while also maximizing control over discretionary revenues. To remain in power they need to maintain the support of a winning coalition of size W. That coalition is drawn from a pool of people with at least a nominal say in choosing leaders and is known as the selectorate (S). Every-

one in society (N) enjoys the benefit of whatever public goods their government provides while only members of the winning coalition benefit from whatever private goods – such as government-tolerated corruption opportunities – the government provides.

Leaders always face threats to their hold on power at least from a political rival, and sometimes from a mass uprising and/or a coup from within their winning coalition. To maintain loyalty, leaders need to offer their supporters at least as many rewards as a rival can credibly offer. In this regard, an established leader has a significant incumbency advantage over her rivals. Should a rival come to power, he is liable to reorganize his coalition of supporters, replacing some coalition members whose support was essential in coming to power with alternatives whom the leader likes more or believes he can trust more. The risk of being displaced from the coalition and how this risk evolves under new leadership lies at the heart of our explanation of political life cycles.

Leaders have affinities or degrees of idiosyncratic loyalty towards potential supporters: they like or trust some more than others. In the original exposition of selectorate theory, it was assumed that upon coming to power these affinities were revealed and so the leader completely reshuffled the coalition immediately after coming to power. The innovation here is to model the revelation of affinities as a gradual learning process and examine the impact of this learning on coalition dynamics and policy provision.

3.1 Model Setup

Consider a polity with a continuum of residents so that the polity's size is N. The political leader, L, faces threats both from within her coalition of W supporters (which we refer to as the deposition or coup risk) and from the masses, M, who seek to replace the existing political order through revolution. We consider an indefinitely repeated game in which leader L forms a coalition of supporters of size W (technically a mass of W supporters since we consider a continuum of people) and rewards these supporters with public (g_t) and private

(z_t) goods. The masses, player M, decide whether or not to rebel. The coalition then decides whether to replace the leader with a rival, and, should they face a revolutionary threat, whether to suppress the masses or take a passive role and tacitly allow the masses to succeed.

The loyalty of coalition members and their willingness to suppress the masses depends upon three factors: 1) the value of immediate policy rewards, that is to say, the level of public and private goods; 2) the leader's performance on all other issues, modeled as a valence shock, θ_t; and 3) the coalition's expectations of being retained and rewarded in future coalitions. This final factor is critical to the coalition dynamics that arise as a leader learns whom she favors.

Each potential coalition member could be a high or low affinity type, α_H and α_L respectively. These affinities reflect how much the leader favors having the person in her coalition for whatever reason (e.g., trust, shared values, shared ethnicity, personality, agreeableness, or for any other idiosyncratic reason). We assume that the leader likes a_0 proportion of the people at the beginning of her tenure. By this we mean that she regards them as high affinity types. However, she is initially uncertain as to who is a high type and who is actually a low type. She gradually learns who is who. To model this learning process we assume that at the end of each period, the leader detects low affinity types with probability q. Rather than retain such low affinity types – that is people she does not favor, like, or trust – she can purge them and replace them with other people, altering the composition of her future coalitions. As this learning process is repeated, low affinity types are weeded out of the coalition and the proportion of high affinity types within the coalition increases.

For technical convenience, the model assumes that the coalition is composed on a mass of supporters rather than discrete individuals.[1] This assumption raises a number of tech-

[1]The infinitely continuous number of coalition members allows for the use of population statistics which greatly simplifies the exposition. If the coalition were composed of a large finite number of supporters, then the results would be substantively similar as the sample statistics converge to the population statistics.

nical issues in the specification of the game that we discuss in the appendix. For ease of language we will often discuss the coalition choices as being made by individuals rather than a continuum.

3.2 Stage Game

The game starts when a new leader comes to power with an initial coalition of supporters of size W. We consider an indefinitely repeated game, indexed by periods $t = 0, 1, 2, ...$, in each of which the leader sets policy, the masses decide whether to rebel or not, and the coalition decides whether to depose the leader and whether to suppress the masses. After these political deposition risks, the leader faces a mortality risk (she might die) and learns about the affinities of her supporters.

1. Policy formation: Leader offers public and private goods, g_t and z_t.

2. Revolution choice: The masses observe the cost of rebellion, K_t, and decide whether to rebel.

3. Deposition choice: Coalition members observe a valence shock θ_t and decide whether to depose the leader.

4. Suppression choice: If there is a revolutionary movement, the coalition must decide whether to suppress it or allow a revolution to succeed.

5. Mortality risk: With probability h the leader lives; with probability $1 - h$ the leader dies.

6. Learning: With probability q the leader identifies the affinity of a low affinity type. Any coalition member identified as a low affinity type will be purged from the coalition and replaced by an alternative selector in future periods.

The valence shocks, θ_t's, are identically independently distributed with distribution $F(x) = Pr(\theta_t \leq x)$. We assume F is strictly concave, continuous and twice differentiable. Throughout we focus on the exponential distribution. The cost of rebellion in period t has distribution $H(x) = Pr(K_t \leq x)$. We focus on the uniform distribution.

3.3 Payoffs

If the leader remains in power, then members of the coalition receive a payoff of $u(g_t, z_t) - \theta_t + y_t$, where $u(g_t, z_t)$ is the value of the public and private goods rewards, θ_t is the valence shock and y_t reflects the net present value of future rewards. We assume that u is additively separable, concave and twice differentiable. If the leader remains in power, then the masses receive $u(g_t, 0)$, the value of the public goods provision. Only coalition members benefit from private goods.

If a revolution occurs and succeeds, then the game ends with the masses receiving a payoff r_M and coalition members receiving a payoff r_W. If a new leader comes to power without a successful revolution, then the game ends, with the masses receiving payoff c_M and the coalition members receiving payoff c_W. If a coalition member is purged from the coalition, then his payoff is γ. Depending on the context, being removed from the coalition might have extreme consequences, such as being sent to the gulag or killed. However, it might be much more innocuous and simply mean no longer benefiting from graft or policy favors. The key assumption is that being purged is worse than remaining in the coalition, which we formally define as $\gamma < c_W$.

The masses pay cost K_t if they rebel. If the coalition suppresses a rebellion and retains the incumbent, then its members pay cost η. If, in the face of a revolution, the coalition replaces the leader, then coalition members pay cost $\frac{\eta}{2}$ to suppress the revolution. In practice revolutionary movements lose much of their impetus when the leader whom the people are rebelling against is replaced. The lower cost for suppressing a revolution after leader

replacement reflects this diminution in revolutionary fervor.[2]

Mass Action	Outcome	Coalition Payoff	Mass Payoff
No Revolution	Leader Retained	$u(g_t, z_t) - \theta_t + y_t$	$u(g_t, 0)$
	Leader Replaced	c_W	c_M
Revolution	Leader Retained	$u(g_t, z_t) - \theta_t + y_t - \eta$	$u(g_t, 0) - K_t$
	Leader Replaced	$c_W - \frac{\eta}{2}$	$c_M - K_t$
	Revolution Wins	r_W	$r_M - K_t$

Table 1: Payoffs

Leaders operate under a budget constraint. They have revenue R, and they provide g_t public and z_t private goods. The cost of public goods is p and the cost of private goods is the number of people who receive them, W, so the cost of providing policy is $pg_t + Wz_t \leq R$.

The incumbent's well-being, or utility, depends on four components: survival in office (Ψ); unspent resources, $(R - pg_t - Wz_t)$; being surrounded by "friends" $(a_t\mu)$, and the cost of purging members from the coalition, $(\varepsilon\,\kappa)$. The first two components are straightforward elements. Leaders want to survive in office (the office holding benefit is Ψ) and use state funds for their own discretionary purposes.[3]

$$U_L(g_t, z_t) = \underbrace{\mathbb{1}_{Survive}}_{Pr(\text{survive})}\Psi + \underbrace{R - pg_t - Wz_t}_{\text{discretionary resources}} + \underbrace{\mu\, a_t}_{\text{among friends}} - \underbrace{\varepsilon\,\kappa}_{\text{cost of purge}}. \tag{1}$$

Leaders like to surround themselves with people they like and trust. a_t refers to the proportion of high affinity types in the coalition in period t. The leader does not want low affinity types, people she dislikes or distrusts, in the coalition. However, it is costly to purge coalition members and so the leader does not want to purge people without due cause. Let κ refer to the cost of replacing a coalition member. Let ε represent the proportion of coalition

[2]The theoretical results, of course, do not depend on this lower suppression cost. We model this feature because we believe it is substantively appropriate.

[3]To focus on interior solutions, we assume that the leader has sufficient resources such that she never exhausts available resources.

members who were replaced from the coalition at the end of the previous period. Then the leader pays replacement cost $\varepsilon\kappa$.

For all analyses, we make these last two factors extremely small: $\mu \to 0$, $\kappa \to 0$ and $\frac{\kappa}{\mu} \to 0$. We do so because we do not want to be concerned with idiosyncratic factors related to leaders, but rather we want to focus on the impact that shuffling coalition membership has on the provision of policy. The leader's desire to be among "friends" means that if the leader detects that any coalition members are of low affinity, then she wants to replace them. The purge cost means that the leader does not want to replace supporters without good cause, with "good cause" meaning any replaced supporter was detected to be a low affinity type. The net effect of these factors is that leaders replace people when they learn that they are low affinity types but leaders keep all others in the coalition.

Figure 1 depicts the sequence of play as a game tree.

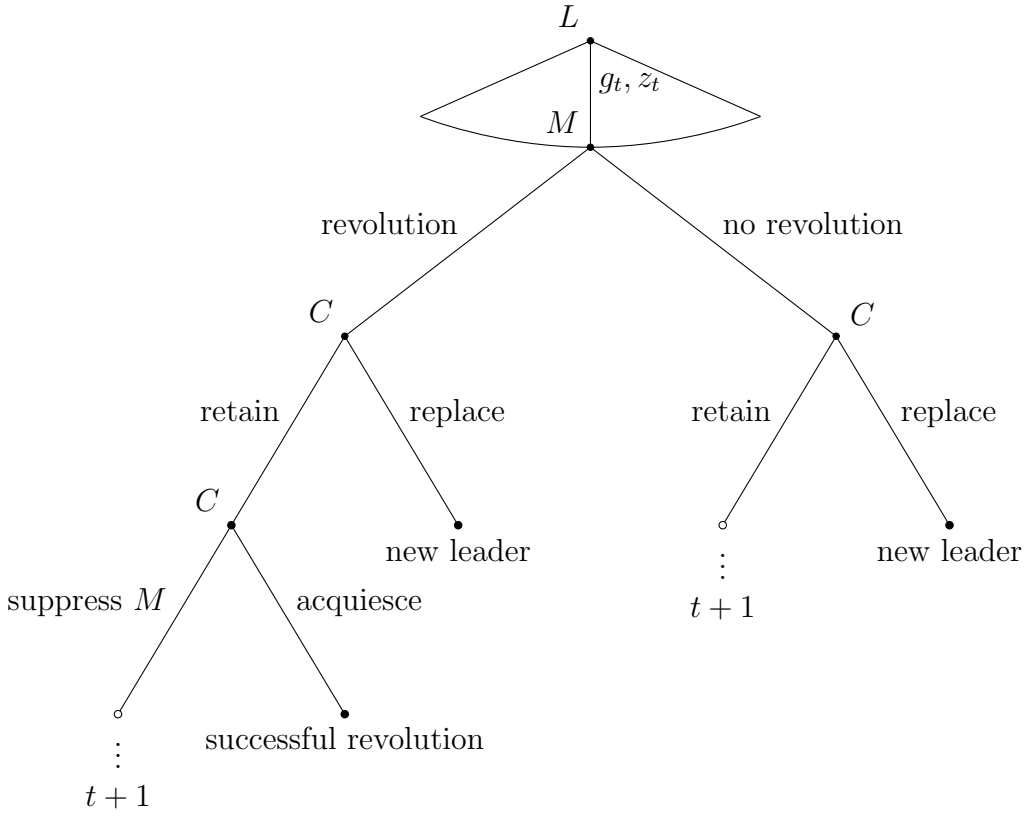


Figure 1: The Game Tree

4 Theoretical Analysis

With the sequence of play and payoffs established, we now turn to the analysis of the game. In doing so, we characterize the unique subgame perfect equilibrium in weakly undominated strategies. Coalition members have a common discount factor, discounting future payoffs by the discount factor δ. We assume the leader and masses maximize payoffs on a period-by-period basis.

4.1 Affinity Technology and Purging

Let ρ_t represent the likelihood that a member of the coalition is purged at the end of period t. The key substantive assumption for our model is that over time the leader learns whom she likes and trusts and whom she dislikes and distrusts. She purges the latter from the coalition. Initially there are many people in the coalition that the leader will eventually want to replace, but over time, as replacement has already taken place, the rate at which the leader purges supporters declines and supporters retained in the coalition have an increasing expectation of being retained in the coalition in the future as it is becoming more densely filled with high affinity types.

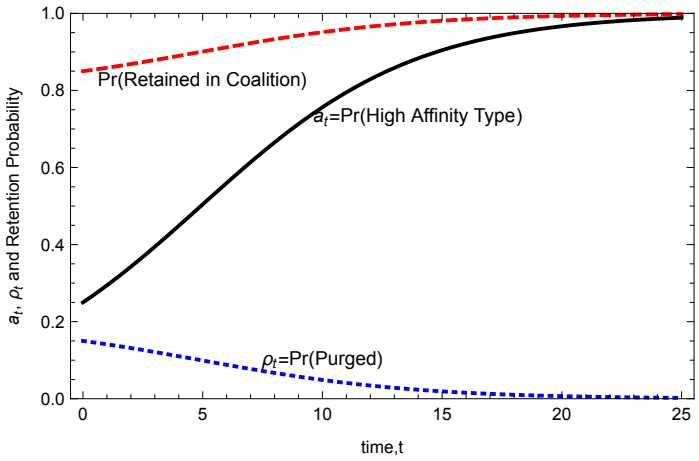
Recall that the initial proportion of people that the leader likes is $a_0 = Pr(\alpha_H)$. In each period there is a chance, q, that the leader identifies a low affinity person: $Pr(\text{identify}|\alpha_L) = q$. The leader learns this information about members of her coalition and potential members (or at least some sufficient subset of selectors). For simplicity, we assume that there are no false positives, $Pr(\text{identify}|\alpha_H) = 0$. Let a_t represent the probability at the start of the period that an unidentified individual in the coalition is a high affinity type. By Bayes's Rule, $a_t = \frac{a_0}{a_0+(1-a_0)Pr(\text{not identified before period t})} = \frac{a_0}{a_0+(1-a_0)(1-q)^t}$. We assume that the friendship relation is unidirectional, so a selector's opinion of the leader does not help him determine whether the leader will like him or not.

The leader purges any coalition member identified as a low affinity type. The probability of being purged at the end of period t is therefore $\rho_t = (1 - a_t)q = \frac{q(1-a_0)(1-q)^t}{a_0+(1-a_0)(1-q)^t}$, which is decreasing in t. Therefore,

Proposition 1 *The probability of being purged from the coalition declines over time. In the limit, as* $t \to \infty$, $\rho_t \to 0$.

Figure 2 graphically illustrates how the proportion of high types in the coalition, the probability of being purged, and the probability that a member is retained in the coalition change over the tenure of a leader.

Figure 2: Learning Dynamics and the Likelihood of Being Purged



To ease the exposition of the game it is useful to introduce notation for the net present value, or continuation value, of the game for coalition members. Let Y_t represent the net present value of being in the coalition at the start of period t. As introduced above, let y_t represent the net present value of coalition membership at the start of step 5 of the stage game; that is, y_t is the net present value of coalition membership after the revolution and political deposition stages have occurred but before the game's mortality risk and learning stages are played out. The continuation value y_t then depends upon three eventualities:

whether the incumbent retains the supporter; whether the incumbent purges the supporter; or whether the leader dies before the start of the next period. The net present value of being retained in the coalition is δY_{t+1}; the net present value of being purged from the coalition is $\delta\gamma$; and the net present value of the new leader for the coalition member following the incumbent's death is δc_W. Therefore

$$y_t = \underbrace{\delta h(1-\rho_t)Y_{t+1}}_{\text{retained in coalition}} + \underbrace{\delta h \rho_t \gamma}_{\text{purged from coalition}} + \underbrace{\delta(1-h)c_W}_{\text{leader dies}}. \tag{2}$$

To clarify, Y_t represents the expected value of the game for a coalition member at the start of period t and y_t represents the continuation value after the political deposition phase of the stage game. With these continuation values in hand, we next derive the occurrence of revolutions, the coalition's choice and the optimal policy allocations by leaders.

4.2 Policy Provisions, Political Survival and Coalition Dynamics

4.2.1 Suppression, Deposition and Revolution

Suppose that in period t the leader provides g_t public goods and z_t private goods. These policies are worth $u(g_t, z_t)$ to coalition members and $u(g_t, 0)$ to the masses. If the masses do not revolt, then the coalition decides whether to retain or depose the leader. If the coalition members depose the leader, then their payoff is c_W. If they retain her, then their payoff is $u(g_t, z_t) - \theta_t + y_t$, where $u(g_t, z_t)$ corresponds to the current period policies, θ_t to the valence shock, and y_t to expectations of future rewards. The coalition members prefer to retain their leader provided that

$$\theta_t \leq \hat{\theta}_t = u(g_t, z_t) + y_t - c_W. \tag{3}$$

That is, the leader is retained if the valence shock (current dissatisfaction with her performance) is smaller than the value she is expected to produce for coalition members (through

the current provision of public and private goods and their continuation value) compared to their expected value if she is replaced.

If a revolution occurs, then the coalition chooses between retaining the leader (worth $u(g_t, z_t) - \theta_t - \eta + y_t$), deposing the leader (worth $c_W - \frac{\eta}{2}$), or allowing the revolution to succeed (worth r_W). There are two cases: in the first case, if $c_W - \frac{\eta}{2} \geq r_W$, then the coalition retains the leader if

$$\theta_t \leq \tilde{\theta}_t = u(g_t, z_t) + y_t - c_W - \frac{\eta}{2}, \tag{4}$$

and deposes her otherwise. In the second case, if $c_W - \frac{\eta}{2} < r_W$, then the coalition suppresses the revolution and retains the leader if

$$\theta_t \leq \overline{\theta_t} = u(g_t, z_t) + y_t - \eta - r_W, \tag{5}$$

otherwise the coalition allows the revolution to succeed.

In case 1, if the masses rebel, then the leader will be deposed if $\theta_t > \tilde{\theta}_t$ which occurs with probability $1 - F\left(\tilde{\theta}_t\right)$. The masses rebel if and only if

$$\underbrace{F\left(\tilde{\theta}_t\right) u(g_t, 0) + \left(1 - F\left(\tilde{\theta}_t\right)\right) c_M - K_t}_{\text{value of rebellion}} \geq \underbrace{F\left(\hat{\theta}_t\right) u(g_t, 0) + \left(1 - F\left(\hat{\theta}_t\right)\right) c_M}_{\text{no rebellion}}.$$

So the masses rebel only if

$$K_t \leq \tilde{k_t} = (c_M - u(g_t, 0)) \left(F\left(\hat{\theta}_t\right) - F\left(\tilde{\theta}_t\right)\right), \tag{6}$$

which occurs with probability $H(\tilde{k})$. Note that in case 1, the masses have no expectation of revolutionary success. However, by rebelling they help precipitate the ouster of the incumbent by the coalition.

In case 2, $c_W - \frac{\eta}{2} < r_W$, the masses rebel if and only if

$$\underbrace{F\left(\overline{\theta_t}\right) u(g_t, 0) + \left(1 - F\left(\overline{\theta_t}\right)\right) r_M - K_t}_{\text{value of rebellion}} \geq \underbrace{F\left(\hat{\theta}_t\right) u(g_t, 0) + \left(1 - F\left(\hat{\theta}_t\right)\right) c_M}_{\text{no rebellion}},$$

so the masses rebel whenever

$$K_t \leq \overline{k_t} = r_M - c_M + F(\hat{\theta}_t)\left(c_M - u(g_t, 0)\right) - F\left(\overline{\theta_t}\right)\left(r_M - u(g_t, 0)\right), \tag{7}$$

which occurs with probability $H(\overline{k})$. Equations 3, 4, 5, 6 and 7 characterize coalition and mass behavior given policies g_t and z_t and expectations of future rewards y_t.

4.2.2 Optimal Policy Provision

If $c_W - \frac{\eta}{2} \geq r_W$ (case 1), then the leader's payoff is

$$\mathcal{L} = \Psi\left(F(\hat{\theta}_t) - H(\tilde{k_t})\left(F(\hat{\theta}_t) - F(\tilde{\theta}_t)\right)\right) + R - p\, g_t - W\, z_t. \tag{8}$$

The first term corresponds to the probability the leader survives and the latter terms are the cost of the policies. Given the assumption that the benefit of being around friends and the cost of purging are small ($\mu \to 0$ and $\kappa \to 0$), these components are omitted. If $c_W - \frac{\eta}{2} < r_W$ (case 2), then there is an analogous expression for $\mathcal{L}$ in which the $\tilde{\theta}$ and $\tilde{k}$ terms are replaced by $\overline{\theta}$ and $\overline{k}$ terms. Maximizing the leader's payoff with respect to g_t and z_t provides a characterization of the leader's optimal policies. To reduce notation, we suppress the subscript t's.

Proposition 2 *In case 1, $c_W - \frac{\eta}{2} \geq r_W$, given y_t expectations about future payoffs L's*

policies g_t and z_t are characterized by

$$\frac{W}{u_z(g,z)\,\Psi} = \underbrace{f(\hat{\theta}) - H(\tilde{k})\left(f(\hat{\theta}) - f(\tilde{\theta})\right) - \left(F(\hat{\theta}) - F(\tilde{\theta})\right) H'(\tilde{k})\left(f(\hat{\theta}) - f(\tilde{\theta})\right)(c_M - u(g_t, 0))}_{X_1} \quad (9)$$

and

$$\frac{p}{u_g(g,z)\,\Psi} = X_1 + \underbrace{\left(F(\hat{\theta}) - F(\tilde{\theta})\right)^2 H'(\tilde{k})}_{X_2}, \quad (10)$$

where $u_z(g,z) = \frac{\partial u(g,z)}{\partial z}$, $u_g(g,z) = \frac{\partial u(g,z)}{\partial g}$ *and* $f = F'$.

The characterization for case 2 is similar and is shown in the appendix along with proofs. Proposition 2 provides clear implications. The LHS of equations 9 and 10 are the marginal costs of policy provision (W with respect to private goods and p with respect to public goods) divided by the product of office holding value and the marginal values of additional policy rewards. The RHS of the equations shows how increased policy rewards reduce the risk of deposition. Two substantively important results follow from these equations.

First, the leader spends additional resources on policy rewards up to the point that her increased likelihood of retaining office equals her marginal cost of providing more policy. Second, relative to most efficiently rewarding her coalition members, the leader's policy provisions are 'biased' towards public goods. Formally, define this public goods bias as $bias = \frac{p}{u_g(g,z)} / \frac{W}{u_z(g,z)} = \frac{X_1 + X_2}{X_1}$. There is an additional term (X_2) on the RHS of equation 10 that is not in equation 9. Providing additional policy benefits buys coalition loyalty that makes coalition members more likely to retain the leader and suppress any revolution. If the leader's only concern were buying coalition loyalty, then she would most efficiently spend resources by balancing marginal costs with marginal benefits: in this case, $bias = 1$. However, public goods enhance survival through a second mechanism. Increased public goods make the

status quo more valuable to the masses and this reduces the likelihood that they will rebel. Since the leader is more likely to be deposed if a revolution occurs ($\tilde{\theta}, \overline{\theta} < \hat{\theta}$), the leader is biased towards public goods as these public rewards restrain revolutionary tendencies while also rewarding supporters.

4.3 Political Life Cycles

Our exposition of the existence and effects of political life cycles relies on the following comparative static result:

Proposition 3 *The comparative statics with respect to* y_t *are* $0 > \frac{dg_t}{dy_t} > -\frac{1}{u_g(g_t,z_t)}$ *and* $0 > \frac{dz_t}{dy_t} > -\frac{1}{u_z(g_t,z_t)}$. *Therefore,* $\frac{du(g_t,z_t)}{dy_t} < 0$ *but* $\frac{d}{dy_t}(y_t + u(g_t, z_t)) > 0$ *so* $\hat{\theta}_t$, $\tilde{\theta}_t$ *and* $\overline{\theta_t}$ *are increasing in* y_t *and* $\tilde{k_t}$ *and* $\overline{k_t}$ *are decreasing in* y_t.

While couched in calculus terms, this result has a simple substantive interpretation. As the value of future coalition membership increases ($y_t \uparrow$), the leader reduces her provision of rewards in the immediate period ($g_t, z_t \downarrow$). However, the extent to which she cuts back on policy provision does not fully offset the increased value of future rewards. The net effect of future coalition membership being more valuable is that the current valence shock thresholds under the different cases ($\tilde{\theta}$, $\overline{\theta}$ and $\hat{\theta}$) all increase, even as the leader provides less policy. The intuition is a straightforward consequence of convex preferences. The leader prefers a mix of increased survival and less spending to a larger increase in survival. If future membership in the coalition, y_t, becomes more valuable, then the coalition is more loyal. In turn, this loyalty deters revolutionaries from attempting an uprising. However, rather than enjoying only the survival benefit of this enhanced loyalty, the leader can slightly cut back on policy provision, keeping slightly more resources for herself and still survive at a higher rate.

The results from propositions 1 and 3 combine to generate the political life cycle result.

Proposition 4 *As leader tenure increases, the leader provides fewer goods, is more likely to survive, faces fewer revolutions, and the bias towards public goods diminishes.*

The life cycle implications follow from a set of changes that we have established take place as a leader's tenure in office increases. Coalition members recognize that their risk of being purged from the coalition decreases the longer they have been in the leader's coalition. This enhanced expectation of being retained in the coalition increases their net present value of being a coalition member. Consequently, this increases the coalition's loyalty to the incumbent. Since the coalition, being more loyal, is less likely to replace the leader or back a revolution, the leader can afford to offer its members fewer rewards. Paradoxically, despite spending less to reward her coalition, the likelihood that the incumbent retains power still increases. Furthermore, as political loyalty grows within the coalition, the masses are more deterred from rebellion. This deterrence diminishes the leader's incentive to favor public goods as a means to buy off the masses and so the proportion of private benefits increases even as total spending to secure the coalition's loyalty decreases over the incumbent's political life cycle.

5 Empirical Analysis

The life cycle selectorate model implies numerous testable hypotheses. Here we focus on those directly related to how long a leader has been in power and what benefits that leader provides. Specifically, we test:

1. The longer a leader has been in office, holding coalition size constant, the fewer rewards s/he provides to members of the winning coalition.

2. The longer a leader has been in office, holding coalition size constant, the fewer public goods s/he provides.

3. The longer a leader has been in office, holding coalition size constant, the greater the proportion of benefits provided in the form of private goods.

4. The length of time a leader has been in office has a significant, but secondary, effect on the provision of benefits as compared to the impact of winning coalition size.

The results also imply that as tenure increases leaders are less likely to be deposed and are less likely to face revolutionary threats from the masses. Given space constraints we do not investigate these additional predictions here.

5.1 Data and Estimation

We test our theoretical predictions with data from 165 countries between 1970 and 2019 across a wide range of policy provisions to document the effects of the political life cycle. We examine three categories of benefits: private goods, public goods, and coordination goods. Many of our policy measures use data from the V-Dem Project (Coppedge et al., 2022).

To measure Private goods we use the V-Dem variable *v2excrptps*, the extent of corruption in the public sector. The basic measure of Public goods is V-Dem's *v2peapspol* variable, which examines equity in provision of public services across political groups. This variable captures the extent to which government policies enhance the welfare of the people in general rather than just political supporters. We also examine Coordination goods. Policies such as freedom of assembly, free press and free speech are of a similar flavor to public goods and represent the extent to which citizens can freely express their views and organize against the government while remaining free from political persecution. We use the *v2caassemb* from V-Dem as the main measure of coordination goods. It measures the extent to which people enjoy freedom of peaceful assembly. Other measures of coordination goods that we investigate include the extent to which press freedom is restricted (Freedom House, 2017), freedom from torture using *v2cltort* from V-Dem, and the political terror scale (Haschke,

2019). To place our measures on similar magnitude scales, we standardize our Private, Public and Freedom of Assembly measures to take values between 0 and 1.

While some of our analyses investigate individual policies or their consequences, we also examine the model's aggregate predictions. Over time the theory anticipates that the leader provides fewer overall rewards. We define Total Rewards as the sum of Public Goods + Private Goods + Freedom of Assembly, where each of the component variables has been standardized to take values between 0 and 1. This aggregate level of rewards is anticipated to decline as tenure increases.

The theory predicts that as tenure increases, leaders spend less on rewarding their coalition and shift the focus of policy provision towards private goods. As a result, the level of public goods should unambiguously decline with tenure. The provision of private goods is more nuanced as it is determined by two offsetting effects: the overall decline in rewards and a shift in the optimal mix of policies towards private goods. Hence, rather than test the effect of tenure on Private goods in isolation, we examine the focus of government policy by looking at the ratio $\frac{Private}{Public+Private}$. This ratio is anticipated to increase as leader tenure increases.

The foundation of the political life cycle is that policy provisions change over the course of a leader's tenure. As such, our main independent variable of interest is the length of the leader's time in office. We utilize data by Smith (2022) which updates the Archigos data (Goemans, Gleditsch and Chiozza, 2009) through December 2021. These data provide the entry and exit time of leaders from office. These data also code the date of death of leaders and, for leaders who die within 10 years of leaving office, the cause of death. We also include data on leader health and relative leader age, which considers age relative to the life expectancy in Sweden. The health variable is dichotomous and codes whether a leader will die of a chronic illness within the next five years (Bueno de Mesquita and Smith, 2018).

Political institutions are, in selectorate logic, the main driver of the types of policies that

leaders provide. The institutional measure that we employ is the magnitude of the size of the winning coalition. We use the new measure of this concept proposed by Bueno de Mesquita and Smith (2022), which places polities on a 0 to 1 scale.

We control for numerous characteristics of each state's economic health. Measures of wealth (per capita GDP in constant $), economic growth, population size and the extent of a nation's natural resources wealth (as a % of GDP) are taken from the World Development Indicators (World Bank, 2022).

Our unit of observation is the country-year. To examine variance in policy provision over time, we estimate a series of fixed effects ordinary least squares (OLS) regression models. Specifically, for a policy outcome Y_{it} for country i in year t, we estimate $Y_{it} = X'_{it}\beta + \lambda_i + \omega_t + \varepsilon_{it}$, where $X'_{it}\beta$ is a vector of covariates, including leader tenure and coalition size. We specify country and year fixed effects, notated by λ_i and ω_t respectively.

5.2 Empirical Results

An ideal test of our hypotheses would examine policy provisions year-by-year throughout the tenure of each leader while holding constant the institutions under which the leader governed. The theoretical exposition, however, tells us that changes to institutions are themselves potentially endogenous to the political life cycle. That is, the institutional structure informs the optimal mix of policy provisions designed to enhance leader survival; leader survival over time further shapes the types of policies leaders will pursue. And, as the policy mix changes, so too does the optimal institutional configuration. Hence, we must acknowledge the inherent difficulty that arises when coalition size changes endogenously.

We could sample only on those polities that we know, *ex post*, did not experience a change in coalition size during a leader's tenure, but this greatly reduces the sample size and relies on information that leaders did not know when they began modifying the benefits they provided to their coalition. Although such an approach yields strongly supportive

results, we prefer to rely only on *ex ante* information and on a sample that is unrestricted. Hence, our tests control for the lagged size of the country's winning coalition, W_{t-1}. This is an imperfect substitute for monitoring leaders under conditions where their institutional settings are unchanging but we believe it is the appropriate and demanding way to test the hypotheses.

Our hypothesis tests are divided into two tables. Table 2 reports the results from our estimating equations for Total Rewards (the sum of Public Goods + Private Goods + Freedom of Assembly), Public Goods Spending, and the proportion of Private Goods to Public Goods. Recall that hypotheses 1 and 2 indicate, respectively, that the coefficient on tenure with respect to Total Rewards and Public Goods should be significantly negative. Hypothesis 3 indicates, conversely, that the coefficient for tenure when the dependent variable is $\frac{Private}{Private+Public}$ should be positive and significant. This is exactly what we see in Table 2.

Table 2 also indicates, as expected, that the larger the polity's winning coalition, the greater is the total spending on benefits; the greater is the provision of public goods; and the smaller is the provision of private goods. For instance, moving from a relatively small to a relatively large coalition system, say $W_{t-1} = .4$ to $W_{t-1} = .8$ (a change from the 25th percentile to the 70th percentile in our data), would lead to an increase of about 0.344 in the provision of Total Rewards, which is approximately one standard deviation of the dependent variable. By contrast, moving from a leader's first to tenth year in office, the comparable change is a decline of policy provision of -0.05, only about 13% of a standard deviation of the dependent variable. While the cross-institutional differences are large (and probably understated as the analyses include country fixed effects and many nations experience only modest institutional change over our period of study), increasing tenure, as hypothesized, significantly reduces the quantity and quality of governance. For public goods, we see that moving from a small to a large coalition system explains about two-fifths of a standard deviation of the dependent variable, while the life cycle effect explains about one-twentieth

of a standard deviation. Similarly, when considering the ratio of private to public goods, a comparable shift in coalition size explains about a quarter of a standard deviation in the dependent variable, while the life cycle effect explains about 5%. Thus, while we see that institutional configurations play a dominant role in describing policy provision, we also see a consistent and significant effect of a leader's tenure in predicting governance outcomes.

	Total Rewards	Public Goods	$\frac{Private}{Private+Public}$
	(1)	(2)	(3)
Log(Tenure)	-0.021***	-0.008**	0.006***
	(0.006)	(0.003)	(0.002)
W_{t-1}	0.860***	0.226***	-0.104***
	(0.075)	(0.033)	(0.025)
Log(GDPpc$_{t-1}$)	0.005	0.033***	-0.031***
	(0.022)	(0.011)	(0.008)
Log(Population$_{t-1}$)	-0.044	-0.013	0.015
	(0.033)	(0.019)	(0.015)
Growth	0.0003	0.0007*	-0.0007**
	(0.0009)	(0.0004)	(0.0003)
Resource Rents	-0.001	-0.001**	0.0010**
	(0.0010)	(0.0005)	(0.0004)
Relative Age	8.59×10^{-5}	-0.0004	0.0002
	(0.0006)	(0.0003)	(0.0002)
Leader Sick	-0.020	-0.006	0.002
	(0.016)	(0.008)	(0.006)
Observations	7,065	7,087	7,087
R^2	0.862	0.932	0.941
Within R^2	0.377	0.208	0.134
Country fixed effects	✓	✓	✓
Year fixed effects	✓	✓	✓

Standard errors clustered by country.
$^{*}p < 0.05,\ ^{**}p < 0.01,\ ^{***}p < 0.001.$

Table 2: Estimation of the Political Life Cycle: Main Results (OLS)

Table 2 is clearly supportive of the model's predictions regarding the effects of a leader's life cycle on the shifting provision of goods. Institutions play a dominant role in determining

policy provisions, but the overall size of policy rewards declines with tenure and a longer tenure is associated with a private goods policy focus. Table 3 probes the results further, turning our attention to the economically inexpensive but politically costly provision of the subset of public goods we have called coordination goods. That is, Table 3 investigates how a leader's life cycle in power influences the provision of Freedom of Assembly, Restricted Press Freedom, and, in the reverse direction, the deprivation of freedoms through the use of Torture and a scaled estimation of the government's use of political terror (PTS).

Freedom of Assembly decreases dramatically as a leader's time in office increases. The same is true of press freedom. Leaders significantly add restrictions on the press the longer they are in power. Torture, as well, finds significantly more use as a leader gets deeper into the political life cycle. Again, we see that cross-sectional effects dominate temporal effects in terms of predicting the provision of freedoms. For our Freedom of Assembly variable, moving from a coalition size of $W_{t-1} = 0.4$ to $W_{t-1} = 0.8$ increases the provisions of freedoms by 1.1 standard deviations of our measure, while increasing tenure from 1 to 10 years decreases the freedom of assembly by about 17% of a standard deviation. Directionally, the same is true when abuse of subjects is measured using the Political Terror Scale but for PTS the effect of the life cycle is statistically insignificant. While the impact of tenure is small relative to the impact of coalition size, the theoretically-derived expectations regarding the existence and impact of a leader life cycle are strongly supported by the empirical evidence.

As robustness tests, we repeated all analyses excluding observations that contained instances of leader change, since one might legitimately argue that such observations should be coded as the first year of a new leader, rather than the last year of the departing leader (as we coded it). Further, we repeated all the policy analyses excluding economic growth as a control, since a leader's policies might partially determine growth. The results were very similar to those reported in the text. Finally, we repeated the analyses using leader fixed effects. Given that the average tenure is around four years, such analyses are extremely

	Freedom of Assembly (1)	Restricting Press (2)	Torture (3)	PTS (4)
Log(Tenure)	-0.019***	1.38***	0.126***	0.016
	(0.004)	(0.431)	(0.030)	(0.028)
W_{t-1}	0.691***	-41.4***	-4.20***	-0.803***
	(0.044)	(4.79)	(0.298)	(0.251)
Log(GDPpc_{t-1})	0.009	-2.00	-0.202*	-0.424***
	(0.014)	(2.14)	(0.120)	(0.117)
Log(Population_{t-1})	-0.073***	5.45*	0.126	0.488***
	(0.024)	(3.18)	(0.168)	(0.173)
Growth	-6.58×10^{-5}	-0.032	-0.007*	-0.012**
	(0.0005)	(0.085)	(0.004)	(0.005)
Resource Rents	-0.0009*	0.037	0.010***	0.004
	(0.0005)	(0.042)	(0.004)	(0.004)
Relative Age	2.1×10^{-5}	-0.117***	0.0001	-0.001
	(0.0003)	(0.037)	(0.003)	(0.003)
Leader Sick	-0.013	1.78	0.082	0.050
	(0.011)	(1.17)	(0.058)	(0.076)
Observations	7,074	3,831	7,096	6,287
R^2	0.890	0.933	0.889	0.691
Within R^2	0.463	0.165	0.413	0.053
Country fixed effects	✓	✓	✓	✓
Year fixed effects	✓	✓	✓	✓

Standard errors clustered by country.
$^{*}p < 0.05,\ ^{**}p < 0.01,\ ^{***}p < 0.001.$

Table 3: Results on Freedoms

demanding in terms of degrees of freedom. The assembly and torture results remained significant. However, for the other policy results, the directional predictions remained the same, but statistical significance dropped below conventional levels.

6 Conclusion

We have proposed a revision of selectorate theory in which leaders gradually replace coalition members as they learn which of their supporters are of low affinity. The revised theory, which also allows for some tradeoff between raising survival prospects and securing greater control over discretionary revenue, leads to several novel hypotheses regarding policy mixes and policy consequences. Four of the novel hypotheses are tested here and are found to be strongly supported by data for 165 countries spanning the years from 1970-2019.

The theory indicates and the evidence shows that there is a pro-public goods bias when leaders initially come to power and that bias fades the longer they are in office. The evidence also demonstrates that total resource allocations intended to maintain coalition loyalty decrease with the leadership life cycle even as coalitional loyalty to the incumbent increases due to the decreased risk of being purged by the incumbent in the future. The shifting provision of benefits leads to an indeterminate effect on the quantity of private rewards given to coalition members but to a predicted – and observed – increase in the coalition's private goods relative to public goods rewards.

7 Appendix

In the exposition of the game we describe the actions of the coalition as if the coalition is a unitary actor; linguistically easy, but not strictly true. There is a mass of individuals within the coalition and the coalition's action is the aggregate of their choices. To formalize

this simply, suppose that the coalition's action depends upon the majority's choice; i.e., the leader is replaced if at least $W/2$ mass of supporters defect and the revolution is suppressed if at least $W/2$ mass of supporters choose to suppress. Since there is a mass of supporters, no individual is pivotal in deciding the coalition decision. By the restriction to weakly undominated strategies, every supporter's choice is their most preferred outcome. Since all coalition members are ex ante identical, all coalition members pick the same action and so the coalition members act in consort. In the main text we spoke of the coalition taking a coordinated action, and, in weakly undominated subgame perfect strategies, the equilibrium behavior of the coalition members is to all act identically.

7.1 Proofs

Proof of Proposition 1: Follows directly from Bayes's Rule. ■

Since the following proofs are messy we will introduce the following abbreviated notation to simplify the statement of first and second order conditions. Let $u_z = u_z(g,z) = \frac{du(g,z)}{dz}$, and $u_g = u_g(g,z) = \frac{du(g,z)}{dg}$ which, by additive separability, also equals $\frac{du(g,0)}{dg}$.

Proof of Proposition 2: The proof follows from the maximization of equation 8 with respect to the leader's policy choice's g_t and z_t. We show the details for case 1. Case 2 is analogous. Equations 9 and 10 are rearrangements of the First Order Conditions: $\frac{d\mathcal{L}}{dz} = 0$ and $\frac{d\mathcal{L}}{dg} = 0$.

$$\begin{aligned}\frac{d\mathcal{L}}{dz} &= -W + \Psi\left[f(\hat{\theta})u_z - H(\tilde{k})(f(\hat{\theta}) - f(\tilde{\theta}))u_z - (F(\hat{\theta}) - F(\tilde{\theta}))H'(\tilde{k})\frac{d\tilde{k}}{dz}\right] \\ &= -W + \Psi u_z\left[\underbrace{f(\hat{\theta}) - H(\tilde{k})(f(\hat{\theta}) - f(\tilde{\theta})) - (F(\hat{\theta}) - F(\tilde{\theta}))H'(\tilde{k})(c_m - u(g,0))(f(\hat{\theta}) - f(\tilde{\theta}))}_{X_1}\right],\end{aligned}$$

where $\frac{d\tilde{k}}{dz} = (c_m - u(g,0))(f(\hat{\theta}) - f(\tilde{\theta}))u_z$.

$$\frac{d\mathcal{L}}{dg} = -p + \Psi u_g \left[X_1 + \underbrace{(F(\hat{\theta}) - F(\tilde{\theta}))^2 H'(\tilde{k})}_{X_2} \right],$$

where the extra term X_2 arises from the differentiation of the $(c_m - u(g,0))$ term in $\tilde{k}$. The SOC are

$$\frac{d^2\mathcal{L}}{dz^2}\frac{1}{\Psi} = u_{zz}X_1 + u_z^2 X_3 < 0,$$

where $u_{zz} = \frac{d^2u(g,z)}{dz^2}$ and X_3 can be thought of as the derivative $\frac{dX_1}{du(g,z)}$, specifically

$$\begin{aligned} X_3 &= \frac{dX_1}{dz}\frac{1}{u_z} = f'(\hat{\theta}) - (f'(\hat{\theta}) - f'(\tilde{\theta}))H(\tilde{k}) - (f(\hat{\theta}) - f(\tilde{\theta}))^2 H'(\tilde{k})(c_M - u(g,0)) \\ &- (f(\hat{\theta}) - f(\tilde{\theta}))^2 H'(\tilde{k})(c_m - u(g,0)) - (f'(\hat{\theta}) - f'(\tilde{\theta}))(F(\hat{\theta}) - F(\tilde{\theta}))H'(\tilde{k})(c_m - u(g,0)) \\ &- (F(\hat{\theta}) - F(\tilde{\theta}))(f(\hat{\theta}) - f(\tilde{\theta}))(c_m - u(g,0)) \underbrace{H''(\tilde{k})}_{\text{0, since H is uniform}} \frac{d\tilde{k}}{du(g,z)} < 0. \end{aligned}$$

$$\frac{d^2\mathcal{L}}{dg^2}\frac{1}{\Psi} = u_{gg}(X_1 + X_2) + u_g^2 \left(X_3 + \underbrace{3(F(\hat{\theta}) - F(\tilde{\theta}))(f(\hat{\theta}) - f(\tilde{\theta}))H'(\tilde{k})}_{X_4} \right) < 0,$$

where the term X_4 is the composite of the derivative of X_2 and the derivative of the $(c_M - u(g,0))$ component of X_1

$$\frac{d^2\mathcal{L}}{dgdz}\frac{1}{\Psi} = \underbrace{u_{zg}}_{0} X_1 + u_z u_g X_3 < 0,$$

where $u_{zg} = \frac{d^2u(g,z)}{dzdg} = 0$ via additive separability. The determinant of the Jacobian is

$$\begin{aligned} |J| &= \begin{vmatrix} \frac{d^2\mathcal{L}}{dg^2} & \frac{d^2\mathcal{L}}{dgdz} \\ \frac{d^2\mathcal{L}}{dgdz} & \frac{d^2\mathcal{L}}{dz^2} \end{vmatrix} \\ &= \Psi^2 \left[u_{zz}u_{gg}X_1^2 + u_{zz}X_1u_g^2(X_3+X_4) + u_{gg}(X_1+X_2)u_z^2X_3 + u_{zz}u_{gg}X_1X_2 + u_z^2u_g^2X_3X_4 \right] > 0. \end{aligned}$$

Given the SOC, $\frac{d^2\mathcal{L}}{dg^2} < 0$, $\frac{d^2\mathcal{L}}{dz^2} < 0$ and $|J| > 0$, for all (g, z). Hence, L's optimization problem is globally concave an the FOC characterize unique globally optimal policies. ■

For case 2, $r_W > c_W - \eta/2$, the relevant FOC that characterize optimal policies are

$$\frac{W}{u_z(g,z)\,\Psi} = \underbrace{f(\hat{\theta}) - H(\overline{k})\left(f(\hat{\theta}) - f(\overline{\theta})\right) - \left(F(\hat{\theta}) - F(\overline{\theta})\right)H'(\overline{k})\left(f(\hat{\theta})\left(c_M - u(g_t,0)\right) - f(\overline{\theta})\left(r_M - u(g_t,0)\right)\right)}_{X_5} \quad (11)$$

and

$$\frac{p}{u_g(g,z)\,\Psi} = X_5 + \left(F(\hat{\theta}) - F(\overline{\theta})\right)^2 H'(\overline{k}) \quad (12)$$

and proof is analogous.

Proof of Proposition 3: Via Cramer's rule,

$$\frac{dg}{dy} = -\frac{\begin{vmatrix} \frac{d^2\mathcal{L}}{dgdy} & \frac{d^2\mathcal{L}}{dgdz} \\ \frac{d^2\mathcal{L}}{dzdy} & \frac{d^2\mathcal{L}}{dz^2} \end{vmatrix}}{|J|} \text{ and } \frac{dz}{dy} = -\frac{\begin{vmatrix} \frac{d^2\mathcal{L}}{dg^2} & \frac{d^2\mathcal{L}}{dgdy} \\ \frac{d^2\mathcal{L}}{dgdz} & \frac{d^2\mathcal{L}}{dzdy} \end{vmatrix}}{|J|},$$

where $|J|$ is the determinant of the Jacobian of SOC (signed in Proposition 2) and

$$\frac{d^2\mathcal{L}}{dgdy} = \Psi u_z X_3 \text{ and } \frac{d^2\mathcal{L}}{dzdy} = \Psi u_g(X_3 + 2(F(\hat{\theta}) - F(\tilde{\theta}))(f(\hat{\theta}) - f(\tilde{\theta}))H(\tilde{k})).$$

Therefore the comparative statics of interest have the following form, $\frac{dg}{dy} = -\frac{A}{Au_g+B}$ where $A, B > 0$ since the all the terms that appear in $\frac{d^2\mathcal{L}}{dgdy}\frac{d^2\mathcal{L}}{dz^2} - \frac{d^2\mathcal{L}}{dgdz}\frac{d^2\mathcal{L}}{dzdy}$ are $\frac{1}{u_g}$ multiples of terms in $|J|$ and $|J|$ contains additional terms. Therefore, $\frac{dg}{dy} \in (0, \frac{1}{u_z})$. Likewise $\frac{dz}{dy}$ takes a similar form, so $\frac{dz}{dy} \in (0, \frac{1}{u_z})$.

Furthermore, since cutoffs in θ_t are in linear in $u(g_t, z_t)$ and y_t, we have $\frac{du(g_t,z_t)}{dy_t} < 0$ but $\frac{d}{dy_t}(y_t + u(g_t, z_t)) > 0$, hence θ_t increasing in y_t. An analogous argument follows for cutoffs in k_t. ■

To proceed examine the continuation value at the beginning of period t:

$$\begin{aligned} Y_t &= \int_0^{\tilde{\theta}_t} (u(g_t, z_t) + y_t - \theta - \eta H(\tilde{k_t}))f(\theta)d\theta \\ &+ (1 - H(\tilde{k_t})) \int_{\tilde{\theta}_t}^{\hat{\theta}_t} (u(g_t, z_t) + y_t - \theta)f(\theta)d\theta + H(\tilde{k_t})(F(\hat{\theta}_t) - F(\tilde{\theta}_t))(c_W - \frac{\eta}{2}) \\ &+ (1 - F(\hat{\theta}_t))c_W, \end{aligned} \tag{13}$$

where by differentiation of equation 13, $\frac{dY_t}{dy_t} \in (0, 1)$.

Since $y_t = \delta(1-\rho_t)hY_{t+1} + \delta h\rho\gamma + \delta(1-h)$, $\frac{dy_t}{dY_{t+1}} = \delta(1-\rho_t)h$, $\frac{dy_t}{d\rho_t} = -h\delta(Y_{t+1} - \gamma) < 0$ and $\frac{dy_t}{dh} = \delta((1-\rho_t)Y_{t+1} + \rho_t\gamma) > 0$. Therefore, $0 < \frac{dY_t}{dY_{t+1}} < \delta(1-\rho_t)h < 1$, $\frac{dY_t}{dh} > 0$ and $0 > \frac{dY_t}{d\rho_t} > -h\delta(Y_{t+1} - \gamma)$.

Lemma 1 *For any Y_{t+1}, ρ_t and h there is a unique solution for Y_t.*

Proof of Lemma 1: Consider the RHS of equation 13. The RHS is increasing in Y_{t+1}. As $Y_{t+1} \to -\infty$, the coalition always replaces the leader so $RHS \to c_W$. As $Y_{t+1} \to \infty$, then coalition always stays loyal so $\frac{RHS}{Y_{t+1}} \to \delta h$. Hence the RHS crosses the 45 degree line. Further, since $0 < \frac{dY_t}{dY_{t+1}} < \delta(1-\rho_t)h < 1$, the crossing can occur only once. ■

Proof of Proposition 4: There is a lower bound on Y_t of c_W because the coalition could

always replace the leader and obtain payoff of c_W. Via Proposition 1, as $t \to \infty$, $\rho_t \to 0$ so $y_t = \delta h Y_{t+1} + \delta(1-h)c_W$ and $Y_{t+1} \to Y_t$. As $t \to \infty$, Y_t is defined recursively as the unique solution to

$$\begin{aligned} Y_t &= \frac{1}{1-\delta(1-h)F(\tilde{\theta}) - H(\tilde{k_t})\delta(1-h)(F(\hat{\theta}-F(\tilde{\theta}))} \left[\int_0^{\tilde{\theta_t}} (u(g_t,z_t) + \delta(1-h) - \theta - \eta H(\tilde{k_t}))f(\theta)d\theta \right. \\ &+ (1-H(\tilde{k_t})) \int_{\tilde{\theta_t}}^{\hat{\theta_t}} (u(g_t,z_t) + \delta(1-h)_t - \theta)f(\theta)d\theta + H(\tilde{k_t})(F(\hat{\theta_t}) - F(\tilde{\theta_t}))(c_W - \frac{\eta}{2}) \\ &+ \left. (1-F(\hat{\theta_t}))c_W \right]. \end{aligned} \tag{14}$$

Such a unique solution exists via Lemma 1. Further, for any Y_t, there exists a unique $Y_{t-1} < Y_t$.

Since Y_t increases in t and ρ_t decreases in t, $y_t = \delta h \rho_t \gamma + \delta h(1-\rho_t)Y_{t+1} + \delta(1-h)c_W$ is increasing in t and in h. Via proposition 3, $\hat{\theta}_t, \tilde{\theta}_t$ and $\bar{\theta}$ increase in t and g_t and z_t decrease in t. ■

The proof of Proposition 4 did not explicitly examine how bias changes over time, a topic we now address.

Proof that *bias* **decreases in** t**:** $bias = \frac{p}{u_g(g,z)} / \frac{W}{u_z(g,z)} = \frac{X_1+X_2}{X_1}$. As a simple proof we utilize the distributional functions and write $X_2 = \left(e^{-\hat{\theta}} - e^{-\tilde{\theta}}\right)^2 H'(\tilde{k})$ and writing the uniform distribution $H(\tilde{k})$ as $\tilde{k}H'(\tilde{k})$, $X_1 = e^{-\hat{\theta}} + 2\left(c_M - u(g_t,0)\right)H'(\tilde{k})\left(e^{-\hat{\theta}} - e^{-\tilde{\theta}}\right)^2$ As t increases, $\hat{\theta}$ and $\tilde{\theta}$ get larger and $(c_M - u(g_t,0))$ increases. The $\left(e^{-\hat{\theta}} - e^{-\tilde{\theta}}\right)^2$ terms get small faster than $e^{-\hat{\theta}}$, so X_2 gets smaller faster than X_1. As a result, the bias ratio moves towards 1. ■

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