Online Appendix for:
International Negotiations in the Shadow of Elections

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The appendix contains the formal results in "International Negotiations in the Shadow of Elections." It also contain several results omitted from the main paper. In addition, we present a generalized version of the model in which leaders can negotiate to any point on a continuous policy space.

A Discrete Choice Model

A.1 Setup

We consider a model in which a foreign power F offers a deal to domestic nation in exchange for policy concessions (equivalent to a cost-sharing agreement). If an agreement is signed, each player enjoys a value from the agreement θ_i for $i = F, L, R, M$. In exchange for an agreement, the leader of domestic nation D (either L or R) concedes some policy (costs) $x \in [0, 1]$. The ideal point of F is $x_F = 1$ and the ideal point of domestic players is $x_L = x_R = x_M = 0$. Let $v_F(x)$ represent F's payoff from the policy x, $v_F(x)$ is increasing in x. Likewise $v_L(x) = v_R(x) = v_M(x)$ is the policy payoff of the domestic actor that is decreasing in x.

We limit the number of possible agreements to a discrete set of outcomes: $x_0 \leq x_A < x_B < x_C$. Focusing on an interesting case, F myopically prefers all agreements to no agreement: $\theta_F + v_F(x_i) \geq v_F(x_0)$ for $i = A, B, C$. Likewise the dovish domestic player, L, myopically prefers all agreements to no agreement: $\theta_L + v_L(x_i) \geq v_L(x_0)$ for $i = A, B, C$. In contrast the hawkish domestic player, R, myopically prefers no agreement to x_C: $\theta_R + v_R(x_A) > \theta_R + v_R(x_B) > v_R(x_0) > \theta_R + v_R(x_C)$. Throughout all examples utilize $v_F(x) = -(1-x)^2$ and $v_L(x) = v_R(x) = v_M(x) = -x^2$, $\theta_F \geq 0, \theta_L = \frac{1}{2}, \theta_R = \frac{1}{4}$ and $x_A = 0, x_B = \frac{1}{3}$, and $x_C = \frac{2}{3}$.

We vary θ_M, the median voter valuation according to whether M is dovish and likes all agreement ($\theta_M + v_M(x_C) > v_M(0) = 0$) or M is hawkish and myopically dislikes x_C ($\theta_M + v_M(x_C) < 0$).

Note that although in principle F could offer x_A, this option is dominated by offering x_B so we ignore x_A as an option that F would offer throughout.

A.1.1 Second Period Agreements

Let $\hat{x}_j(x_1)$ be the second period agreement given first period agreement x_1 if leader $j \in \{L, R\}$ is elected. For L, the value of agreement x_2 in the final period is $\theta_L + v_L(x_2)$ and the value of no agreement is $v_L(x_0) = 0$. Hence L would remain in deal x_A, x_B or x_C. In contrast, R would exit agreement x_C but could not credibly exit from x_A or x_B.

Proposition 1 *If the first period agreement is x_1 and leader j is elected, then the second period outcome, $x_2 = \hat{x}_j(x_1)$, is shown by Table 1.*

Table 1: Second Period Agreement $x_2 = \hat{x}_j(x_1)$ given Leader $j \in \{L, R\}$ in Power and the First Period Agreement, x_1

First Period Agreement, x_1	x_0	x_A	x_B	x_C
L wins election	x_C	x_A	x_B	x_C
R wins election	x_B	x_A	x_B	x_A

A.1.2 Payoffs Given x_1

Proposition 1, the first period agreement, and the electoral result uniquely define the second period agreement. Let p be the probability that L is elected. If L is elected with probabilities p_0, p_B and p_C given first period outcomes x_0, x_B and x_C, respectively, then the players' expected payoffs for each outcome are as follows. Note that we weight the second period by $\delta \in (0, 1)$.

L's payoffs for first period outcomes:

$U_L(x_0) = v_L(x_0) + \delta\left(\theta_L + p_0\Psi + p_0 v_L(x_C) + (1 - p_0)v_L(x_B)\right)$

$U_L(x_B) = \theta_L + v_L(x_B) + \delta\left(\theta_L + p_B\Psi + v_L(x_B)\right)$

$U_L(x_C) = \theta_L + v_L(x_C) + \delta\left(\theta_L + p_C\Psi + p_C v_L(x_C) + (1 - p_C)v_L(x_A)\right)$

R's payoffs for first period outcomes:

$U_R(x_0) = v_R(x_0) + \delta\left(\theta_R + (1 - p_0)\Psi + p_0 v_R(x_C) + (1 - p_0)v_R(x_B)\right)$

$U_R(x_B) = \theta_R + v_R(x_B) + \delta\left(\theta_R + (1 - p_B)\Psi + v_R(x_B)\right)$

$U_R(x_C) = \theta_R + v_R(x_C) + \delta\left(\theta_R + (1 - p_C)\Psi + p_C v_R(x_C) + (1 - p_C)v_R(x_A)\right)$

F's payoffs for first period outcomes:

$U_F(x_0) = v_F(x_0) + \delta\left(\theta_F + p_0 v_F(x_C) + (1 - p_0)v_F(x_B)\right)$

$U_F(x_B) = \theta_F + v_F(x_B) + \delta\left(\theta_F + v_F(x_B)\right)$

$U_F(x_C) = \theta_F + v_F(x_C) + \delta\left(\theta_F + p_C v_F(x_C) + (1 - p_C)v_F(x_A)\right)$

A.2 Electoral Probabilities

Let $\bar{p} = G(\beta)$ be the baseline probability that L is elected. As a simplification, assume that density of G is uniform. $G(\beta + \sigma y) = G(\beta) + g\sigma y$ where $g = G'$.

Table 2: Electoral Probabilities that L is Elected

<table>
<tr><td>Electoral Assumption</td><td>Incumbent L</td><td>Incumbent R</td></tr>
<tr><td>Exogenous elections</td><td>$\bar{p} = G(\beta)$</td><td>$\bar{p} = G(\beta)$</td></tr>
<tr><td></td><td colspan="2">Retrospective Voters</td></tr>
<tr><td>x_0
$x_1 = x_B$
$x_1 = x_C$</td><td>$p_0 = G(\beta)$
$p_B = G(\beta + \sigma(\theta_M + v_M(x_B)))$
$p_C = G(\beta + \sigma(\theta_M + v_M(x_C)))$</td><td>$p_0 = G(\beta)$
$p_B = G(\beta - \sigma(\theta_M + v_M(x_B)))$
$p_C = G(\beta - \sigma(\theta_M + v_M(x_C)))$</td></tr>
<tr><td></td><td colspan="2">Prospective Voters</td></tr>
<tr><td>Generic, x_1
x_0
x_B
x_C</td><td colspan="2">$p_{x_1} = G(\beta + \sigma(v_M(\hat{x}_L(x_1)) - v_M(\hat{x}_R(x_1)))$
$p_{x_0} = G(\beta + \sigma(v_M(x_C) - v_M(x_B))$
$p_{x_B} = G(\beta + \sigma(v_M(x_B) - v_M(x_B)) = G(\beta)$
$p_{x_C} = G(\beta + \sigma(v_M(x_C) - v_M(x_A))$</td></tr>
</table>

A.3 Dovish Incumbent

A.3.1 Agreements that L Accepts

It is straightforward to see that L always accepts x_B: myopically he supports the agreement and such an agreement maximizes his probability of election. However, L only accepts x_C if $U_L(x_C) \geq U_L(x_0)$. We define

$$\alpha_C^L = \begin{cases} 1 + \frac{\theta_L + v_L(x_C)}{\delta(v_L(x_A) - v_L(x_B))} + \frac{g\sigma(v_M(x_C) + \theta_M)(-v_L(x_A) + v_L(x_C) + \Psi)}{v_L(x_A) - v_L(x_B)} & \text{if retrospective election} \\ 1 + \frac{\theta_L + v_L(x_C)}{\delta(v_L(x_A) - v_L(x_B))} - \frac{g\sigma\Psi(v_M(x_A) - v_M(x_B))}{v_L(x_A) - v_L(x_B)} & \\ + \frac{g\sigma((v_L(x_A) - v_L(x_C))v_M(x_A) - (v_L(x_B) - v_L(x_C))v_M(x_B) - (v_L(x_A) - v_L(x_B))v_M(x_C))}{v_L(x_A) - v_L(x_B)} & \text{if prospective election} \end{cases} \tag{1}$$

The exogenous election case corresponds to $\sigma = 0$ (removes the final term in each case). If $\bar{p} = G(\beta) \leq \alpha_C^L$ then $U_L(x_C) \geq U_L(x_0)$ and so L accepts a first period offer of x_C. In the exogenous election, or the retrospective election with a dovish median voter ($\theta_M + v_M(x_C) > 0$), then $\alpha_C^L > 1$ so L always accepts x_C. If the election context is retrospective and the median voter is hawkish ($\theta_M + v_M(x_C) < v_M(x_0)$) or if the election context is prospective, then as office holding incentives dominate ($\Psi \to \infty$) L always rejects x_C.

A.3.2 F's Preferred Offer to L

Define ρ_{C0}^L as the probability $\bar{p} = G(\beta)$ such that F is indifferent between the first round deals x_C and x_0. If $\bar{p} = G(\beta) \geq \rho_{C0}^L$, then F prefers x_C to x_0. Define ρ_{B0}^L and ρ_{BC}^L as the analogous indifferences from F's perspective between x_B and x_0 and between x_B and x_C. If $\bar{p} = G(\beta) \geq \rho_{BC}^L$, then F prefers the first round outcome x_C to the first round outcome x_B. If $\bar{p} = G(\beta) \geq \rho_{B0}^L$, then F prefers the first round outcome x_0 to the first round outcome x_B.

$$\rho^L_{C0} = \begin{cases} 1 - \frac{\theta_F + v_F(x_C) - v_F(x_0)}{\delta(v_F(x_B) - v_F(x_A))} - \frac{g\sigma(\theta_M + v_M(x_C))(v_F(x_C) - v_F(x_A))}{v_F(x_B) - v_F(x_A)} & \text{if retrospective} \\ 1 + \frac{\theta_F - v_F(x_0) + v_F(x_C)}{\delta v_F(x_A) - \delta v_F(x_B)} & \\ + \frac{g\sigma(v_F(x_C)(-v_M(x_A) + v_M(x_B)) + v_F(x_A)(v_M(x_A) - v_M(x_C)) + v_F(x_B)(-v_M(x_B) + v_M(x_C)))}{v_F(x_A) - v_F(x_B)} & \text{if prospective} \end{cases} \tag{2}$$

$$\rho^L_{B0} = \begin{cases} \frac{\theta_F + v_F(x_B) - v_F(x_0)}{\delta v_F(x_C) - \delta v_F(x_B)} & \text{if retrospective} \\ -\frac{\theta_F - v_F(x_0) + v_F(x_B)}{\delta v_F(x_B) - \delta v_F(x_C)} + g\sigma\left(v_M\left(x_B\right) - v_M\left(x_C\right)\right) & \text{if prospective} \end{cases} \tag{3}$$

$$\rho^L_{BC} = \begin{cases} -\frac{v_F(x_C) + \delta v_F(x_A) - (1+\delta) v_F(x_B)}{\delta(v_F(x_C) - v_F(x_A))} - g\sigma\left(\theta_M + v_M\left(x_C\right)\right) & \text{if retrospective} \\ \frac{\delta v_F(x_A) - (1+\delta) v_F(x_B) + v_F(x_C)}{\delta(v_F(x_A) - v_F(x_C))} + g\sigma\left(v_M\left(x_A\right) - v_M\left(x_C\right)\right) & \text{if prospective} \end{cases} \tag{4}$$

The exogenous election conditions are given when $\sigma = 0$. Given the definitions α^L_C, ρ^L_{C0}, ρ^L_{B0} and ρ^L_{BC}, we can characterize subgame perfect equilibrium outcomes. For instance, x_C is the SPE outcome when L will accept x_C, that is to say $\bar{p} \leq \alpha^L_C$, and F prefers the outcome x_C to x_B and x_0 ($\bar{p} \geq \rho^L_{BC}$ and $\bar{p} \geq \rho^L_{C0}$). The proposition follows directly from simple logical statements about which of the acceptable proposals F most prefers:

Proposition 2 *If L is the incumbent, then the first period deal is*

$$x_1 = \begin{cases} x_B & \textit{if } \bar{p} \leq \rho^L_{B0} \textit{ and (either } \bar{p} \leq \rho^L_{BC} \textit{ or } \bar{p} \geq \alpha^L_C\textit{)} \\ x_C & \textit{if } \bar{p} \leq \alpha_{LC0} \textit{ and } \bar{p} \geq \rho^L_{C0} \textit{ and } \bar{p} \geq \rho^L_{BC} \\ x_0 & \textit{if } (\bar{p} \geq \alpha_{LC0} \textit{ or } \bar{p} \leq \rho^L_{C0}) \textit{ and } \bar{p} \geq \rho^L_{B0} \end{cases} \tag{5}$$

These conditions define the thresholds between the regions shown in the figures throughout the paper. We next examine how the thresholds shift in response to changes in different parameters.

Proposition 3 *The sign of the comparative statics of how the thresholds change with respect to the parameters are given in Table 3 for both retrospective and prospective election settings. For threshold y and parameter z, the table provides the sign of $\frac{dy}{dz}$. The hawk and dove references refers to the median voter (specifically, $\theta_M + v_M(x_C) < 0$ implies hawk in the retrospective context).*

For a prospective election and a retrospective election with hawkish voters: As Ψ, σ, or δ increase, L is more likely to reject x_C. In the retrospective case (with hawkish voters), increases in θ_M increases the likelihood that L accepts x_C.

The value of office holding for D does not affect F's preferences over deals, although increases in Ψ reduce the parameters for which L will accept x_C.

Table 3: Comparative Statics for the thresholds with dovish incumbent (L) with respect to Ψ, σ, θ_M and δ.

Threshold/parameter	Retrospective Election				Prospective Election			
	Ψ	σ	θ_M	δ	Ψ	σ	θ_M	δ
α_C^L	-	-	+	-	-	-	0	-
ρ_{C0}^L	0	+hawk, -dove	-	+	0	?, depends on parameters	0	+
ρ_{B0}^L	0	0	0	-	0	+	0	-
ρ_{BC}^L	0	+hawk, -dove	-	+	0	+	0	+

A.4 Hawkish Incumbent

A.4.1 Agreements that R Accepts

Analogous to the approach above, define α_C^R such that if $\bar{p} \leq \alpha_C^R$ then R prefers the first period agreement x_C rather than waiting (x_0) i.e. $\bar{p}$ such that $U_R(x_C) \geq U_R(x_0)$. Likewise define α_B^R such that if $\bar{p} \geq \alpha_B^R$ then R prefers the first period agreement x_B rather than waiting (x_0) i.e. $\bar{p}$ such that $U_R(x_B) \geq U_R(x_0)$. This condition is always satisfied in the retrospective election context.

$$\alpha_C^R = \begin{cases} 1 + \frac{\theta_R + v_R(x_C) - v_R(x_0)}{\delta(v_R(x_A) - v_R(x_B))} + \frac{g\sigma(\theta_M + v_M(x_C))(\Psi + v_R(x_A) - v_R(x_C))}{v_R(x_A) - v_R(x_B)} & \text{if retrospective election} \\ 1 + \frac{\theta_R - v_R(x_0) + v_R(x_C)}{\delta(v_R(x_A) - v_R(x_B))} & \\ -g\sigma v_M(x_C) + \frac{g\sigma(v_M(x_A)(\Psi + v_R(x_A) - v_R(x_C)) - v_M(x_B)(\Psi + v_R(x_B) - v_R(x_C)))}{v_R(x_A) - v_R(x_B)} & \text{if prospective election} \end{cases} \tag{6}$$

$$\alpha_B^R = \begin{cases} 0 & \text{if retrospective election} \\ \frac{-\theta_R + v_R(x_0) - v_R(x_B)}{\delta(v_R(x_B) - v_R(x_C))} + \frac{g\sigma(v_M(x_B) - v_M(x_C))(\Psi + v_R(x_B) - v_R(x_C))}{v_R(x_B) - v_R(x_C)} & \text{if prospective election} \end{cases} \tag{7}$$

A.4.2 F's Preferred Offer to R

We define ρ_{B0}^R as the value of $\bar{p} = G(\beta)$ such that F is indifferent between the first period outcome x_B and no first period agreement, x_0. Likewise ρ_{C0}^R and ρ_{BC}^R define F's indifference between x_C and x_0 and x_B and x_C, respectively.

$$\rho^R_{B0} = \begin{cases} \frac{\theta_F - v_F(x_0)+v_F(x_B)}{\delta(v_F(x_C)-v_F(x_B))} & \text{if retrospective election} \\ \frac{\theta_F - v_F(x_0)+v_F(x_B)}{\delta(v_F(x_C)-v_F(x_B))} + g\sigma\left(v_M\left(x_B\right) - v_M\left(x_C\right)\right) & \text{if prospective election} \end{cases} \tag{8}$$

$$\rho^R_{C0} = \begin{cases} 1 + \frac{\theta_F - v_F(x_0)+v_F(x_C)}{\delta(v_F(x_A)-v_F(x_B))} - \frac{g\sigma(v_F(x_A)-v_F(x_C))(\theta_M+v_M(x_C))}{v_F(x_B)-v_F(x_A)} & \text{if retrospective election} \\ 1 - \frac{\theta_F - v_F(x_0)+v_F(x_C)}{\delta(v_F(x_B)-v_F(x_A))} & \\ + \frac{g\sigma(v_F(x_C)(v_M(x_A)-v_M(x_B))-v_F(x_A)(v_M(x_A)-v_M(x_C))+v_F(x_B)(v_M(x_B)-v_M(x_C)))}{v_F(x_B)-v_F(x_A)} & \text{if prospective election} \end{cases} \tag{9}$$

$$\rho^R_{BC} = \begin{cases} \frac{(1+\delta)v_F(x_B)-\delta v_F(x_A)-v_F(x_C)}{\delta(v_F(x_C)-v_F(x_A))} + g\sigma\left(\theta_M + v_M\left(x_C\right)\right) & \text{if retrospective election} \\ \frac{-\delta v_F(x_A)+(\delta+1)v_F(x_B)-v_F(x_C)}{\delta(v_F(x_C)-v_F(x_A))} + g\sigma\left(v_M\left(x_A\right) - v_M\left(x_C\right)\right) & \text{if prospective election} \end{cases} \tag{10}$$

Given the definitions α^R_C, α^R_B, ρ^R_{B0}, ρ^R_{B0} and ρ^R_{BC}, we can characterize subgame perfect equilibrium outcomes. For instance, x_C is the SPE outcome when R will accept x_C, that is to say $\bar{p} \leq \alpha^R_C$ and F prefers the outcome x_C to x_0 ($\bar{p} \geq \rho^L_{C0}$) and (either F prefers x_C to x_B ($\bar{p} > \rho^R_{BC}$) or R rejects x_B ($\bar{p} < \alpha^R_B$)). The proposition follows directly from simple logical statements about which of the acceptable proposals F most prefers:

Proposition 4 *If R is the incumbent, then the first period deal is*

$$x_1 = \begin{cases} x_C & \textit{if } \bar{p} \leq \alpha^R_C \textit{ and } \bar{p} \geq \rho^R_{C0} \textit{ and (either } \bar{p} \geq \rho^R_{BC} \textit{ or } \bar{p} \leq \alpha^R_B\textit{)} \\ x_B & \textit{if } \bar{p} \geq \alpha^R_B \textit{ (which is alway true in the retrospective setting or as } \Psi \to 0\textit{)} \\ & \textit{and } \bar{p} \leq \rho^R_{B0} \textit{ and (either } \bar{p} \leq \rho^R_{BC} \textit{ or } \bar{p} \geq \alpha^R_C\textit{)} \\ x_0 & \textit{if (either } \bar{p} \geq \rho^R_{B0} \textit{ or } \bar{p} \leq \alpha^R_B\textit{) and (either } \bar{p} \leq \rho^R_{C0} \textit{ or } \bar{p} \geq \alpha^R_C\textit{)} \end{cases} \tag{11}$$

These conditions define the thresholds between the regions shown in the figures throughout the paper. We next examine how the thresholds shift in response to parameters.

Proposition 5 *The sign of the comparative statics of how the thresholds change with respect to the parameters are given in Table 4 for both retrospective and prospective election settings. For threshold y and parameter z, the table provides the sign of $\frac{dy}{dz}$. The hawk and dove references refers to the median voter (specifically, $\theta_M + v_M(x_C) < 0$ implies hawk).*

Table 4: Comparative Statics for the thresholds with hawkish incumbent (R) with respect to Ψ, σ, θ_M and δ.

Threshold/parameter	Retrospective Election				Prospective Election			
	Ψ	σ	θ_M	δ	Ψ	σ	θ_M	δ
ρ^R_{B0}	0	0	0	-	0	+	0	-
ρ^R_{C0}	0	-hawk, +dove	+	+	0	?, depends on parameters	0	+
ρ^R_{B0}	0	-hawk, +dove	+	+	0	+	0	+
α^R_C	-hawk, +dove	-hawk, +dove	+	+	+	+	0	+
α^R_B					+	+	0	+

A.5 Additional Proofs

Proof of Proposition 3: Comparative statics of α^L_C: Retrospective election context:

$$\begin{aligned}
\frac{d\alpha^L_C}{d\Psi} &= \frac{g\sigma\left(v_M\left(x_C\right)+\theta_M\right)}{v_L\left(x_A\right)-v_L\left(x_B\right)}<0\\
\frac{d\alpha^L_C}{d\sigma} &= \frac{g\left(v_M\left(x_C\right)+\theta_M\right)\left(-v_L\left(x_A\right)+v_L\left(x_C\right)+\Psi\right)}{v_L\left(x_A\right)-v_L\left(x_B\right)}<0\\
\frac{d\alpha^L_C}{d\theta_M} &= \frac{g\sigma\left(-v_L\left(x_A\right)+v_L\left(x_C\right)+\Psi\right)}{v_L\left(x_A\right)-v_L\left(x_B\right)}>0\\
\frac{d\alpha^L_C}{d\delta} &= -\frac{v_L\left(x_C\right)+\theta_L}{\delta^2\left(v_L\left(x_A\right)-v_L\left(x_B\right)\right)}<0
\end{aligned}$$

Comparative statics of α^L_C: Prospective election context:

$$\begin{aligned}
\frac{d\alpha^L_C}{d\Psi} &= \frac{g\sigma\left(v_M\left(x_B\right)-v_M\left(x_A\right)\right)}{v_L\left(x_A\right)-v_L\left(x_B\right)}<0\\
\frac{d\alpha^L_C}{d\sigma} &= g\left(\frac{\left(v_M\left(x_B\right)-v_M\left(x_A\right)\right)\left(v_L\left(x_C\right)+\Psi\right)}{v_L\left(x_A\right)-v_L\left(x_B\right)}+\frac{v_L\left(x_A\right)v_M\left(x_A\right)-v_L\left(x_B\right)v_M\left(x_B\right)}{v_L\left(x_A\right)-v_L\left(x_B\right)}-v_M\left(x_C\right)\right)\\
&\quad \text{which is negative for large } \Psi\\
\frac{d\alpha^L_C}{d\theta_M} &= 0\\
\frac{d\alpha^L_C}{d\delta} &= -\frac{v_L\left(x_C\right)+\theta_L}{\delta^2\left(v_L\left(x_A\right)-v_L\left(x_B\right)\right)}<0
\end{aligned}$$

Comparative statics of ρ^L_{C0}: In the retrospective election context:

$$\begin{aligned}
\frac{d\rho^L_{C0}}{d\Psi} &= 0\\
\frac{d\rho^L_{C0}}{d\sigma} &= -\frac{g\left(v_F\left(x_A\right)-v_F\left(x_C\right)\right)\left(v_M\left(x_C\right)+\theta_M\right)}{v_F\left(x_A\right)-v_F\left(x_B\right)} + \text{ for hawk voter, - for dove voters}\\
\frac{d\rho^L_{C0}}{d\theta_M} &= -\frac{g\sigma\left(v_F\left(x_A\right)-v_F\left(x_C\right)\right)}{v_F\left(x_A\right)-v_F\left(x_B\right)}<0\\
\frac{d\rho^L_{C0}}{d\delta} &= \frac{v_F\left(x_C\right)+\theta_F-v_F\left(x_0\right)}{\delta^2\left(v_F\left(x_B\right)-v_F\left(x_A\right)\right)}>0
\end{aligned}$$

Comparative statics of ρ_{C0}^{L}: In the prospective election context:

$$\begin{aligned}
\frac{d\rho_{C0}^{L}}{d\Psi} &= 0\\
\frac{d\rho_{C0}^{L}}{d\sigma} &= \frac{g\left(v_F\left(x_C\right)\left(v_M\left(x_B\right)-v_M\left(x_A\right)\right)+v_F\left(x_A\right)\left(v_M\left(x_A\right)-v_M\left(x_C\right)\right)+v_F\left(x_B\right)\left(v_M\left(x_C\right)-v_M\left(x_B\right)\right)\right)}{v_F\left(x_A\right)-v_F\left(x_B\right)}\\
\frac{d\rho_{C0}^{L}}{d\theta_M} &= 0\\
\frac{d\rho_{C0}^{L}}{d\delta} &= -\frac{v_F\left(x_C\right)+\theta_F-v_F\left(x_0\right)}{\delta^2\left(v_F\left(x_A\right)-v_F\left(x_B\right)\right)}>0
\end{aligned}$$

Comparative statics of ρ_{BC}^{L}: In the retrospective election context:

$$\begin{aligned}
\frac{d\rho_{BC}^{L}}{d\Psi} &= 0\\
\frac{d\rho_{BC}^{L}}{d\sigma} &= -g\left(v_M\left(x_C\right)+\theta_M\right)\ \text{+ for hawk voters, - for dove voters}\\
\frac{d\rho_{BC}^{L}}{d\theta_M} &= -g\sigma<0\\
\frac{d\rho_{BC}^{L}}{d\delta} &= \frac{v_F\left(x_B\right)-v_F\left(x_C\right)}{\delta^2\left(v_F\left(x_A\right)-v_F\left(x_C\right)\right)}>0
\end{aligned}$$

Comparative statics of ρ_{BC}^{L}: In the prospective election context:

$$\begin{aligned}
\frac{d\rho_{BC}^{L}}{d\Psi} &= 0\\
\frac{d\rho_{BC}^{L}}{d\sigma} &= g\left(v_M\left(x_A\right)-v_M\left(x_C\right)\right)>0\\
\frac{d\rho_{BC}^{L}}{d\theta_M} &= 0\\
\frac{d\rho_{BC}^{L}}{d\delta} &= \frac{v_F\left(x_B\right)-v_F\left(x_C\right)}{\delta^2\left(v_F\left(x_A\right)-v_F\left(x_C\right)\right)}>0
\end{aligned}$$

Comparative statics of ρ_{B0}^{L}: In the retrospective election context:

$$\begin{aligned}
\frac{d\rho_{B0}^{L}}{d\Psi} &= 0\\
\frac{d\rho_{B0}^{L}}{d\sigma} &= 0\\
\frac{d\rho_{B0}^{L}}{d\theta_M} &= 0\\
\frac{d\rho_{B0}^{L}}{d\delta} &= -\frac{\left(v_F\left(x_C\right)-v_F\left(x_B\right)\right)\left(v_F\left(x_B\right)+\theta_F-v_F\left(x_0\right)\right)}{\left(\delta v_F\left(x_C\right)-\delta v_F\left(x_B\right)\right)^2}<0
\end{aligned}$$

Comparative statics of ρ_{B0}^{L}: In the prospective election context:

$$\begin{aligned}
\frac{d\rho_{B0}^{L}}{d\Psi} &= 0 \\
\frac{d\rho_{B0}^{L}}{d\sigma} &= g\left(v_M\left(x_B\right) - v_M\left(x_C\right)\right) > 0 \\
\frac{d\rho_{B0}^{L}}{d\theta_M} &= 0 \\
\frac{d\rho_{B0}^{L}}{d\delta} &= \frac{\left(v_F\left(x_B\right) - v_F\left(x_C\right)\right)\left(v_F\left(x_B\right) + \theta_F - v_F\left(x_0\right)\right)}{\left(\delta v_F\left(x_B\right) - \delta v_F\left(x_C\right)\right)^2} > 0
\end{aligned}$$

■

Proof of Proposition 5: Comparative statics of α_B^R: Retrospective election: R always accepts x_B with retrospective voters. Comparative statics of α_B^R: Prospective election:

$$\begin{aligned}
\frac{d\alpha_B^R}{d\Psi} &= \frac{g\sigma\left(v_M\left(x_B\right) - v_M\left(x_C\right)\right)}{v_R\left(x_B\right) - v_R\left(x_C\right)} > 0 \\
\frac{d\alpha_B^R}{d\sigma} &= \frac{g\left(v_M\left(x_B\right) - v_M\left(x_C\right)\right)\left(v_R\left(x_B\right) - v_R\left(x_C\right) + \Psi\right)}{v_R\left(x_B\right) - v_R\left(x_C\right)} > 0 \\
\frac{d\alpha_B^R}{d\theta_M} &= 0 \\
\frac{d\alpha_B^R}{d\delta} &= \frac{v_R\left(x_B\right) + \theta_R - v_R\left(x_0\right)}{\delta^2\left(v_R\left(x_B\right) - v_R\left(x_C\right)\right)} > 0
\end{aligned}$$

Comparative statics of ρ_{B0}^{R}: Retrospective election:

$$\begin{aligned}
\frac{d\rho_{B0}^{R}}{d\Psi} &= 0 \\
\frac{d\rho_{B0}^{R}}{d\sigma} &= 0 \\
\frac{d\rho_{B0}^{R}}{d\theta_M} &= 0 \\
\frac{d\rho_{B0}^{R}}{d\delta} &= -\frac{v_F\left(x_B\right) + \theta_F - v_F\left(x_0\right)}{\delta^2\left(v_F\left(x_C\right) - v_F\left(x_B\right)\right)} < 0
\end{aligned}$$

Comparative statics of ρ^R_{B0}: Prospective election:

$$
\begin{aligned}
\frac{d\rho^R_{B0}}{d\Psi} &= 0 \\
\frac{d\rho^R_{B0}}{d\sigma} &= g\left(v_M\left(x_B\right) - v_M\left(x_C\right)\right) > 0 \\
\frac{d\rho^R_{B0}}{d\theta_M} &= 0 \\
\frac{d\rho^R_{B0}}{d\delta} &= -\frac{v_F\left(x_B\right) + \theta_F - v_F\left(x_0\right)}{\delta^2\left(v_F\left(x_C\right) - v_F\left(x_B\right)\right)} < 0
\end{aligned}
$$

Comparative statics of ρ^R_{C0}: Retrospective election:

$$
\begin{aligned}
\frac{d\rho^R_{C0}}{d\Psi} &= 0 \\
\frac{d\rho^R_{C0}}{d\sigma} &= -\frac{g\left(v_F\left(x_A\right) - v_F\left(x_C\right)\right)\left(v_M\left(x_C\right) + \theta_M\right)}{v_F\left(x_B\right) - v_F\left(x_A\right)} \\
\frac{d\rho^R_{C0}}{d\theta_M} &= -\frac{g\sigma\left(v_F\left(x_A\right) - v_F\left(x_C\right)\right)}{v_F\left(x_B\right) - v_F\left(x_A\right)} > 0 \\
\frac{d\rho^R_{C0}}{d\delta} &= -\frac{v_F\left(x_C\right) + \theta_F - v_F\left(x_0\right)}{\delta^2\left(v_F\left(x_A\right) - v_F\left(x_B\right)\right)} > 0
\end{aligned}
$$

Comparative statics of ρ^R_{C0}: Prospective election:

$$
\begin{aligned}
\frac{d\rho^R_{C0}}{d\Psi} &= \\
\frac{d\rho^R_{C0}}{d\sigma} &= \frac{g\left(v_F\left(x_C\right)\left(v_M\left(x_A\right) - v_M\left(x_B\right)\right) - v_F\left(x_A\right)\left(v_M\left(x_A\right) - v_M\left(x_C\right)\right) + v_F\left(x_B\right)\left(v_M\left(x_B\right) - v_M\left(x_C\right)\right)\right)}{v_F\left(x_B\right) - v_F\left(x_A\right)} \\
\frac{d\rho^R_{C0}}{d\theta_M} &= 0 \\
\frac{d\rho^R_{C0}}{d\delta} &= \frac{v_F\left(x_C\right) + \theta_F - v_F\left(x_0\right)}{\delta^2\left(v_F\left(x_B\right) - v_F\left(x_A\right)\right)} > 0
\end{aligned}
$$

Comparative statics of ρ^R_{BC}: Retrospective election:

$$
\begin{aligned}
\frac{d\rho^R_{BC}}{d\Psi} &= \\
\frac{d\rho^R_{BC}}{d\sigma} &= g\left(v_M\left(x_C\right) + \theta_M\right) \\
\frac{d\rho^R_{BC}}{d\theta_M} &= g\sigma > 0 \\
\frac{d\rho^R_{BC}}{d\delta} &= \frac{v_F\left(x_B\right) - v_F\left(x_C\right)}{\delta^2\left(v_F\left(x_A\right) - v_F\left(x_C\right)\right)} > 0
\end{aligned}
$$

Comparative statics of ρ_{BC}^R: Prospective election:

$$\begin{aligned}
\frac{d\rho_{BC}^R}{d\Psi} &= 0 \\
\frac{d\rho_{BC}^R}{d\sigma} &= g\left(v_M\left(x_A\right) - v_M\left(x_C\right)\right) > 0 \\
\frac{d\rho_{BC}^R}{d\theta_M} &= 0 \\
\frac{d\rho_{BC}^R}{d\delta} &= \frac{v_F\left(x_B\right) - v_F\left(x_C\right)}{\delta^2\left(v_F\left(x_A\right) - v_F\left(x_C\right)\right)} > 0
\end{aligned}$$

Comparative statics of α_C^R: Retrospective election:

$$\begin{aligned}
\frac{d\alpha_C^R}{d\Psi} &= \frac{g\sigma\left(v_M\left(x_C\right) + \theta_M\right)}{v_R\left(x_A\right) - v_R\left(x_B\right)} \\
\frac{d\alpha_C^R}{d\sigma} &= \frac{g\left(v_M\left(x_C\right) + \theta_M\right)\left(v_R\left(x_A\right) - v_R\left(x_C\right) + \Psi\right)}{v_R\left(x_A\right) - v_R\left(x_B\right)} \\
\frac{d\alpha_C^R}{d\theta_M} &= \frac{g\sigma\left(v_R\left(x_A\right) - v_R\left(x_C\right) + \Psi\right)}{v_R\left(x_A\right) - v_R\left(x_B\right)} \\
\frac{d\alpha_C^R}{d\delta} &= -\frac{v_R\left(x_C\right) + \theta_R - v_R\left(x_0\right)}{\delta^2\left(v_R\left(x_A\right) - v_R\left(x_B\right)\right)} > 0
\end{aligned}$$

Comparative statics of α_C^R: Prospective election:

$$\begin{aligned}
\frac{d\alpha_C^R}{d\Psi} &= \frac{g\sigma\left(v_M\left(x_A\right) - v_M\left(x_B\right)\right)}{v_R\left(x_A\right) - v_R\left(x_B\right)} > 0 \\
\frac{d\alpha_C^R}{d\sigma} &= \frac{g\left(v_M\left(x_A\right)\left(v_R\left(x_A\right) - v_R\left(x_C\right) + \Psi\right) - v_M\left(x_B\right)\left(v_R\left(x_B\right) - v_R\left(x_C\right) + \Psi\right)\right)}{v_R\left(x_A\right) - v_R\left(x_B\right)} - g v_M\left(x_C\right) > 0 \text{ for large } \Psi \\
\frac{d\alpha_C^R}{d\theta_M} &= 0 \\
\frac{d\alpha_C^R}{d\delta} &= -\frac{v_R\left(x_C\right) + \theta_R - v_R\left(x_0\right)}{\delta^2\left(v_R\left(x_A\right) - v_R\left(x_B\right)\right)} > 0
\end{aligned}$$

■

B Continuous Choice Model

In the main text we examined a model where the bargains were restricted to a discrete set of outcomes. Here we reformulate the model in a context in which nations can offer any position on a continuous policy space. This continuous version of the model offers the same basic insights as the discrete bargaining version as well as some additional nuanced findings, although it does so at the cost of considerable mathematical complexity.

B.1 Setup

As with the discrete model, within nation D there are two leaders/parties, labeled L and R. In the first period F can offer D's incumbent leader (L or R) an agreement. If D's leader accepts, then the agreement is formed. Then an election occurs in D in which a median voter M picks a leader to govern in the second period. In this second period if there is no existing alliance, then F can offer the leader in D (who may or may not be the same leader) a policy-security agreement and D's leader can accept or reject. If there is an existing alliance, then D's leader can demand a revision to the terms of the agreement. F can accept or reject this revision or terminate the alliance. In the final move D's leader can terminate the alliance. The structure of game ensures that no leader is party to an alliance agreement unless he or she wants to be part of the agreement.

For ease of language we stick to presenting the model in the form of F offering a security alliance to D and we use a specific functional form. However, the model is readily adapted to other settings. The model considers two dimensions: security $s \in \{0, s_A\}$ and policy $x \in [0, 1]$. s reflects the extent to which F enhances D's security and x reflects the extent to which F influences D's foreign policy. F wants to maximize its control of D's policy while minimizing it security responsibilities. In contrast, D wants to enhance its security while maintaining as much discretion as possible over policy. To reflect diminishing marginal returns on each dimension, we model actors' preferences with a simple Cobb-Douglas utility function. Given the policy-security pair $z = (x, s)$, nation F's policy-security payoff is

$$V_F(z) = x^{1-\alpha_F}(1-s)^{\alpha_F}$$

where $\alpha_F \in (0, 1)$ is F's relative salience for security and policy.

The policy-security payoffs of domestic nation actors $i = L, R, M$ are

$$V_i(z) = (1-x)^{1-\alpha_i} s^{\alpha_i}$$

where $\alpha_i \in (0, 1)$ reflects the extent to which i is willing to give up policy in exchange for increased security. We assume that $\alpha_L > \alpha_R$. This is to say, for an increase in security, leader L would be willing to give up more policy than would leader R.

Absent an agreement, the policy-security outcome is the status quo, $z_0 = (x_0, s_0)$. Throughout the paper we utilize the running example shown in Figure 1 in which $z_0 = (\frac{1}{5}, \frac{1}{5})$ and $\alpha_F = \frac{1}{2}$, $\alpha_L = \frac{2}{3}$, $\alpha_R = \frac{1}{3}$ and $\alpha_M = \frac{1}{2}$.

The three curves shown in Figure 1 reflect the indifference curves for F, R and L. Foreign nation F's ideal point is $z = (1, 0)$ and the solid curve shows the set of policy security pairs $z = (x, s)$ that make F indifferent to the status quo, z_0: i.e. $V_F(z) = V_F(z_0)$. The pair $z_F = (x_F, s_A)$ is the intersection between F's indifference curve and security level provided by an alliance. In particular, if x_F is the policy component of the policy-security deal (x_F, s_A), then F would be indifferent this alliance and no agreement.

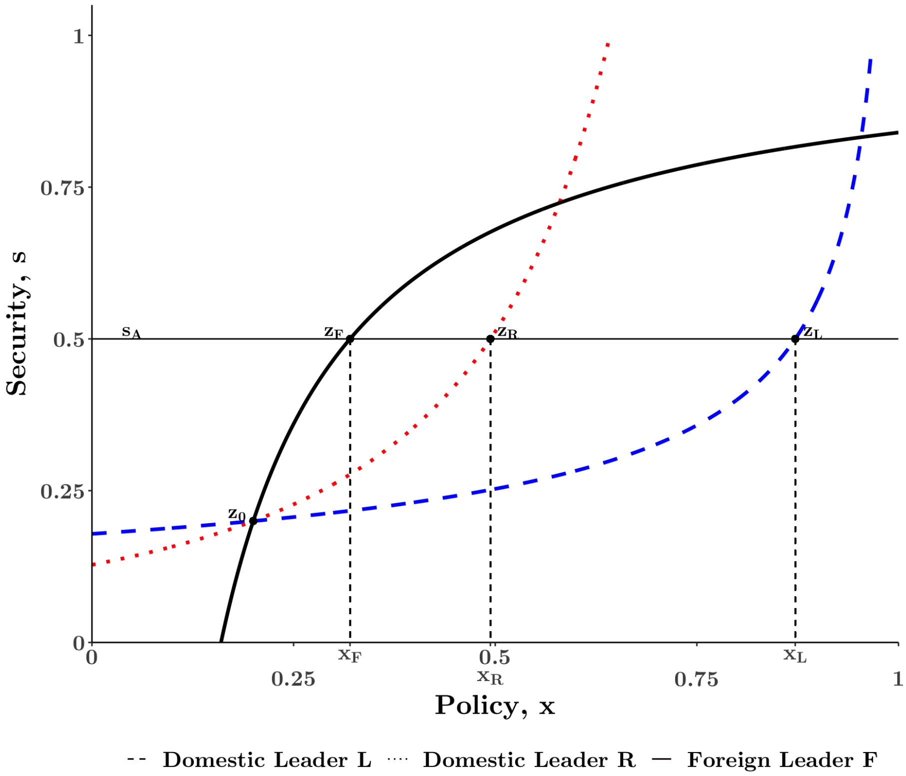


Figure 1: Policy-Security Space and Players' Indifference Curves

The dotted and dashed curves show the analogous indifference curves for leaders R and L and the policies x_R and x_L are the levels of policy concession that make these respective leaders indifferent between a policy-security agreement and no alliance. Given the assumption that L is the more dovish leader ($\alpha_L > \alpha_R$), he is prepared to make the larger policy concession to attain an alliance: $x_L > x_R$. We focus on the case $x_F < x_R < x_L$ shown in Figure 1, in which, myopically, F and R will agree to an alliance provided the policy concession is $x \in [x_F, x_R]$. F and L will agree to a wider range of policy concessions: $x \in [x_F, x_L]$.

We can calculate these indifference values analytically:

$$\begin{aligned} x_F &= \left(\frac{1-s_0}{1-s_A}\right)^{\frac{\alpha_F}{1-\alpha_F}} \\ x_R &= 1-\left(\frac{s_0}{s_A}\right)^{\frac{\alpha_R}{1-\alpha_R}}(1-x_0) \\ x_L &= 1-\left(\frac{s_0}{s_A}\right)^{\frac{\alpha_L}{1-\alpha_L}}(1-x_0) \end{aligned} \tag{1}$$

which, in our running example, come out to $(x_F, x_R, x_L) = (0.32, 0.49, 0.87)$.

Equation 1 characterizes the bounds on deals to which nations would myopically agree. The timeline of the game form is as follows:

1. First Period Policy-Security Negotiation:
   (a) F can demand policy concession $x_1 \in [0, 1]$ in exchange for security agreement s_A.
   (b) D's leader (L or R) either accepts F's proposal creating new status quo $z_1 = (x_1, s_A)$, or rejects, maintaining status quo, $z_1 = z_0$.
2. Election: M selects either L or R as second period leader.
3. Second Period Policy-Security (Re)negotiation:
   - If there is no existing alliance, $z_1 = z_0$:
     (a) F can demand policy concession $x_2 \in [0, 1]$.
     (b) D's leader (L or R) either accepts, $z_2 = (x_2, s_A)$, or rejects, $z_2 = z_0$.
   - If there is an existing alliance, $z_1 \neq z_0$:
     (a) Domestic leader (L or R) offers a renegotiation, x_2.
     (b) F either accepts D's offer ($z_2 = (x_2, s_A)$), retains the existing agreement ($z_2 = (x_1, s_A)$), or exits the policy-security agreement ($z_2 = z_0$).
     (c) Leader D either remains in the agreement ($z_2 = (x_2, s_A)$) or D exits the agreement ($z_2 = z_0$).

Players' payoffs are a weighted sum of the payoffs in each period, where δ reflects the relative importance of post-electoral outcomes. In addition to policy concerns, leaders value office holding, $\Psi > 1$. $\mathbb{I}_{D_2=i}$

is an indicator of whether leader $i = L, R$ is leader of D in the second period. Given the policy-security deals z_1 and z_2 for each period, the actors receive payoffs as follows:

$$U_F(z_1, z_2) = V_F(z_1) + \delta V_F(z_2)$$
$$U_L(z_1, z_2) = V_L(z_1) + \delta(V_L(z_2) + \mathbb{I}_{D_2=L}\ \Psi)$$
$$U_R(z_1, z_2) = V_R(z_1) + \delta(V_R(z_2) + \mathbb{I}_{D_2=R}\ \Psi)$$

B.1.1 Elections

We examine elections on two dimensions: the *salience* of the security-policy dimension to the voters (σ) and whether the voters evaluate leaders in a retrospective or prospective manner ($\rho \in \{0, 1\}$) (Ferejohn 1986). We call this element the *electoral context*.

Voters care about more than just the security-policy trade-off. Let u_L and u_R represent the voters' payoff from L's and R's alliance-policy choices. After observing the first period outcome z_1, voters observe random variables ε_L and ε_R that represent their expectations about the value of L's and R's leadership on all other dimensions.

Let $U_M(\text{elect } L) = \sigma u_L + \beta + \varepsilon_L$ be the median voter's payoff from electing L, where β represents any bias in favor of L on all non-alliance issues and σ is the salience of the security-policy dimension. Let $U_M(\text{elect } R) = \sigma u_R + \varepsilon_R$ be M's payoff from electing R. The median voter thus prefers L to R when

$$\varepsilon = \varepsilon_R - \varepsilon_L \leq \beta + \sigma(u_L - u_R)$$

Let $\varepsilon \sim G$, such that the probability that L is elected is $p = G[\beta + \sigma(u_L - u_R)]$. We assume G has the standard nice properties of being twice differentiable and having full support. Let g be the associated density function.

Retrospective and Prospective Voters

Retrospective voters, $\rho = 1$, make a simple comparison of what the incumbent leader delivered relative to the status quo. If L is the incumbent, then $u_L = V_M(z_1) - V_M(z_0)$, which is the difference in their welfare between a first period agreement and the status quo, and $u_R = 0$. In contrast, if R is the incumbent, then $u_R = V_M(z_1) - V_M(z_0)$ and $u_L = 0$. It is clear that retrospective voters favor an incumbent who signs a deal such that $x_1 \leq x_M$, the point at which the median voter is indifferent between making an alliance and the status quo. Retrospective voters evaluate their leaders of the basis of what they have already delivered.

Conversely, prospective voters base their assessment on what they expect leaders can deliver in the second period. Prospective voters, $\rho = 0$, compare their expected welfare under both leaders in the second period given the agreement reached in the first period. Let the notation $z_L^*(z_1)$ represent the second period alliance-policy outcome given L is elected and the first round outcome is z_1. Likewise $z_R^*(z_1)$ corresponds to the second period outcome if R is elected. Prospective voters evaluate leaders in terms of what they can deliver in the future: $u_L = V_M(z_L^*(z_1))$ and $u_R = V_M(z_R^*(z_1))$.

We provide a general solution and then focuses on four limiting cases. First, we consider myopic leaders ($\delta = 0$) who care only about the immediate period's policy. Second, we consider *exogenous* elections by supposing the voters' electoral choices are unaffected by security-policy outcomes, $\sigma \to 0$. Thirdly and fourthly we consider retrospective and prospective voters in the context that office holding is the dominant concern of leaders, $\Psi \to \infty$.

To focus on substantively interesting cases we impose assumptions on the value of office holding and the sensitivity of elections to alliance outcomes.

Assumption 1 *Office holding is sufficiently valuable that neither leader in D would prefer that the other lead:* $\Psi > 1$.

We also assume that alliance outcomes do not have an overwhelming effect on election outcomes:

Assumption 2 $\delta\sigma g \leq \frac{1-\alpha_F}{1-\alpha_M} \frac{(1-x_1)^{\alpha_M}(1-s_A)^{\alpha_F}}{x_1^{\alpha_F} s_A^{\alpha_M}}$ *for* $x_1 \in [x_R, x_L]$

This assumption states that the product of patience, policy salience and probability density is not too high. Effectively, the assumption rules out small shifts in alliance policy resulting in huge shifts in which leader is elected. Although we believe this a substantively appropriate assumption, since alliance policy is rarely the dominant issue in elections, we primarily impose this assumption for presentational convenience. If this condition does not hold and elections are extremely sensitive to alliance outcomes, then the equilibria are very similar but their characterization requires further conditions since F might forgo additional concessions to which leaders in D would agree in order to alter electoral outcomes.[1]

We categorize the unique subgame perfect equilibrium.

B.2 How Bargains Shape Future Negotiations

The first period deal and election result uniquely determine the policy outcome in the second period (Lemma 1).

The results are summarized in Table 1. Recall $z_L^*(z_1)$ and $z_R^*(z_1)$ denote the second period outcomes under L and R's leadership given first round agreement z_1. Pre-existing agreements shape negotiations.

[1]See discussion following Lemma 4.

D_2	No Alliance, $z_1 = (x_0, s_0)$ x_0	Alliance, $z_1 = (x_1, s_A)$ $x_1 < x_F$	$x_1 \in [x_F, x_R]$	$x_1 \in (x_R, x_L]$	$x_1 > x_L$
Leader L: $z_L^*(z_1)$	z_L	z_F	z_1	z_1	z_F
Leader R: $z_R^*(z_1)$	z_R	z_F	z_1	z_F	z_F

Table 1: Second Period Agreements

B.3 Alliance Agreements in the Shadow of Elections

As summarized in Table 1, the first period deal affects the second period outcome. Thus when negotiating the first period agreement, leaders must consider more than just the immediate policy implications.

The general results depend upon the incumbent domestic leader. The structure of our analysis is as follows: we partition the policy space into three regions, $x_1 \in [x_F, x_R]$, $x_1 \in (x_R, x_L]$, and $x_1 \in (x_L, 1]$. Which region a deal occurs in determines the longevity of an alliance-policy agreement. If $x_1 \in [x_F, x_R]$, then the deal persists whichever leader is elected. If $x_1 \in (x_R, x_L]$, then the deal persists if L is elected, but R would renegotiate the agreement, $z_2 = z_F$. If $x_1 \in (x_L, 1]$, then neither leader would sustain the deal and both would renegotiate to $z_2 = z_F$. There are never any agreements in the range $x_1 < x_F$ (Lemma 2).

Our intention is to focus on substantive implications and to do so we introduce some nomenclature that we formally define in the proofs. Let $\widetilde{x}_L$ represent the largest (first period) policy deal in $[x_F, x_R]$ that L would accept. Let $\widehat{x}_L$ represent the largest policy deal in $(x_R, x_L]$ that L would accept (if L will accept any deal in this region). Finally, let $\overline{x}_L$ represent the largest policy deal in $(x_L, 1]$ that L would accept (if any exists). Likewise define $\widetilde{x}_R$, $\widehat{x}_R$ and $\overline{x}_R$ as analogous terms with reference to the largest deal that R would accept in these regions.

Table 2 describes how the deal in each of these regions affects second period deals and the probability of L being elected. The braces indicate how a first period deal affects L's electoral prospects relative to $G[\beta]$, the probability that L is elected if the alliance agreement plays no role ($\sigma = 0$).

If L is the incumbent, then the possible outcomes that F could obtain in the first round are z_0, $\widetilde{z}_L = (\widetilde{x}_L, s_A)$, $\widehat{z}_L = (\widehat{x}_L, s_A)$ or $\overline{z}_L = (\overline{x}_L, s_A)$. Analogous outcomes are defined for R. F's payoff from any deal z_1 is:

$$U_F(z_1) = V_F(z_1) + \delta Pr(L \text{ elected}) V_F(z_L^*(z_1)) + \delta(1 - Pr(L \text{ elected})) V_F(z_R^*(z_1))$$

F will propose the deal that yields it the highest payoff. Propositions 1 and 2 formalize these results. We now consider more substantive situations to concretize some of the more general findings.

First Period Outcome x_1	z_1	Second Period Outcome L elected, $z_L^*(z_1)$	R elected, $z_R^*(z_1)$	Retrospective Voting $\Pr(L \text{ elected}),\ \rho = 1$	Prospective Voting $\Pr(L \text{ elected}),\ \rho = 0$
No deal, x_0	z_0	z_L	z_R	$G[\beta]$	$\underbrace{G\left[\beta + \sigma(V_M(z_L) - V_M(z_R))\right]}_{-}$
$x_1 \in [x_F, x_R]$	$\widetilde{z}$	z_1	z_1	$\overbrace{G\left[\beta + \sigma(V_M(z_1) - V_M(z_0))\right]}^{+,\ \text{Incumbent } L}$ $\underbrace{G\left[\beta + \sigma(V_M(z_0) - V_M(z_1))\right]}_{-,\ \text{Incumbent } R}$	$G[\beta]$
$x_1 \in (x_R, x_L]$	$\widehat{z}$	z_1	z_F	$\overbrace{G\left[\beta + \sigma(V_M(z_1) - V_M(z_0))\right]}^{+ \text{ if } x_1 < x_M,\ - \text{ else , L incumbent}}$ $\underbrace{G\left[\beta + \sigma(V_M(z_0) - V_M(z_1))\right]}_{+ \text{ if } x_1 < x_M,\ - \text{ else , R incumbent}}$	$\underbrace{G\left[\beta + \sigma(V_M(z_1) - V_M(z_F))\right]}_{-}$
$x_1 \in (x_L, 1]$	$\overline{z}$	z_F	z_F	$\overbrace{G\left[\beta + \sigma(V_M(z_1) - V_M(z_0))\right]}^{-\ \text{Incumbent } L}$ $\underbrace{G\left[\beta + \sigma(V_M(z_0) - V_M(z_1))\right]}_{+\ \text{Incumbent } R}$	$G[\beta]$

Table 2: Electoral and Future Policy Implications of First Period Outcomes

B.4 Limiting Cases

To highlight the important substantive results we focus on a series of limiting cases that isolate the impact of different factors.

B.4.1 Myopic Leaders

Myopic leaders only care about deals in the immediate period, $\delta = 0$. This setting is equivalent to a single shot version of the game, see Corollary 2.

B.4.2 Exogenous Elections

Politicians care about the agreement in both periods ($\delta > 0$) but the first round agreement does not affect the election outcome $p = G[\beta]$, which we formally model as $\sigma = 0$.

Incumbent L

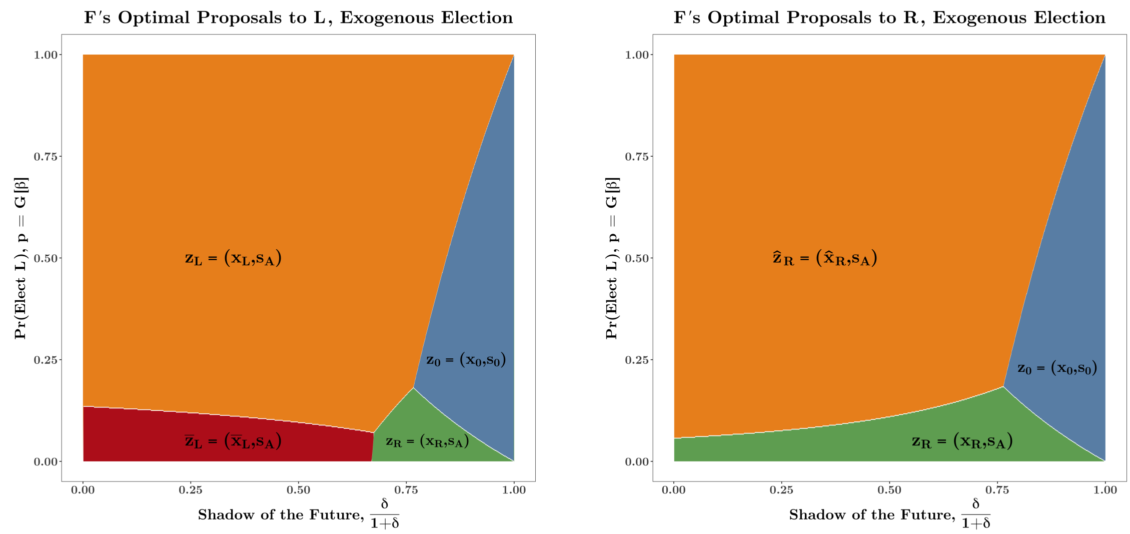
When L is the incumbent, the first round policy will be one of four possible deals: x_0, x_R, x_L or $\overline{x}_L > x_L$. We discuss conditions likely to result in each of these outcomes.

In the myopic case the outcome is always z_L so this seems a natural starting point. If F offers z_L then it retains this outcome in both periods if L wins. However, should R win, the agreement will be renegotiated to z_F. Therefore F's expected payoff is $V_F(z_L) = (1 + p\delta) + \delta(1 - p)V_F(z_F)$. If L is likely to win the election and/or the election is far off (δ small), then extracting the maximum sustainable concession from L is F's best option.

Unfortunately for F, exploiting L's dovishness is less desirable when the election is close (large δ), especially if R is likely to win (small p). An election victory for R leads to renegotiation to a much less desirable deal, z_F. Faced with an immediate election and significant prospects that R will win, F might well prefer either to offer z_R or no alliance, z_0. The deal z_R is sustained whoever wins the election and is attractive especially if R is likely to win. Alternatively, F could form no alliance and then exploit the winner as much as possible after the election. This option is the most preferred when the election is imminent. These arguments are illustrated in first panel of Figure 2.

When the election is far off then F maximally exploits L's dovishness (orange region). As the election casts a bigger shadow and R is likely to win the election, F offers z_R – a deal sustained whoever wins (green region). When the election is imminent, and L has a some chance of winning, then F simply waits and exploits the winner as best it can (blue region).

There is one additional possibility, $\overline{z}_L$, such that $\overline{x}_L > x_L$. Such a deal gives F a great deal today at the expense of getting z_F after the election. It also implies that L gets a poor payoff today in exchange for a great deal in the future. This agreement requires a precise balance so that both L and F prefer the combination of very good and very bad outcomes to something else. In our example, instances of this equilibrium exist (red region), however, F's payoff in this setting is only marginally larger than simply offering z_L. We believe the need for such finely balanced combinations of really good and really poor outcomes for both parties makes such equilibria substantively unimportant in the context of exogenous electoral outcomes. However as we shall see, such behavior is a common feature under prospective voting.



(a) Incumbent L

(b) Incumbent R

Figure 2: First Period Deals as a Function of Relative Importance of the Future (δ) and Exogenous Election Probability (p).

Incumbent R

In the single shot game, F would offer z_R, which R would accept. In the exogenous election setting, F can obtain this same agreement in both periods, and indeed, when R is the likely winner of the election this is exactly what F does (as shown by the green region in the second panel of Figure 2). However, by offering z_R in the first period, F forgoes the opportunity to exploit L's dovishness should L win the election.

If an election is imminent, then F forgoes a first round deal, $z_1 = z_0$, and following the election F fully exploits the winner, with the second period outcome being z_L or z_R depending on who wins the election (blue region). By waiting, F avoids tying its hands and gives it the option of exploiting L's dovishness. Yet, waiting is a poor choice when an election is far off as the surplus from cooperation goes unrealized.

Although R would not myopically agree to a deal $\widehat{z}_R$, such at $\widehat{x}_R > x_R$, she might agree to such a deal in the exogenous electoral setting. If no deal is struck in the first period, then, should L win the election, F will exploit L's dovishness and the second period outcome becomes $z_2 = z_L$. By agreeing to $\widehat{x}_R \in (x_R, x_L)$, R prevents F from maximally exploiting L's dovishness after the election. Further, should R win election, R can obtain her most preferred agreement, z_F, in the second period. When the election is reasonably far off, F prefers to extract extra concessions from R in the short-term, although it does so at the cost of inferior second period outcomes (orange region). And R is willing to make the extra concessions to ensure that F can not exploit L after the election.

It is important to note that although the agreement that F can extract from R exceeds the myopic agreement z_R, F does not extract as much policy from R as when L is the incumbent (i.e. $\widehat{x}_R < x_L$). Although similar regions are colored orange in both panels, in the second panel $x_1 = \widehat{x}_R \in (x_R, x_L)$, while in the first panel $x_1 = x_L > \widehat{x}_R$.

Summary: In the exogenous electoral setting, alliance agreements do not affect who wins the election. Yet the election still influences bargaining as deals struck before the election determine renegotiations after the election.

1. When an election is imminent, F simply waits to see who wins the election (z_0) and then extracts the best deal it can from the winner. Such a bargaining approach results in the loss of a surplus from a deal before the election, but avoids tying F's hands.

2. When R is expected to win the election, F proposes the deal (z_R) sufficient to buy off the hawk. Neither party can renegotiate after the election.

3. Outside of these contingencies, F extracts more from dovish L (z_L) than hawkish R ($\widehat{z}_R$). Both R and L might be willing to make additional short run concessions (relative to myopic agreements) in order to ensure themselves better post-election outcomes.

The presence of an election affects bargaining. Both sides are cognizant that agreements reached today might be renegotiated after the election, so today's deal factors into potential renegotiations.

B.4.3 Agreements and Office-Holding Incentives

If voters care about the security-policy tradeoff ($\sigma > 0$), then pre-election deals affect their vote choices. We examine retrospective and prospective electoral settings. Since the previous section characterized how leaders incorporate the implications of future policy outcomes into their bargaining strategies, here we focus primarily on how the incentive to win elections affects bargaining. To do so we assume office holding is the dominant incentive for domestic leaders by examining equilibria as $\Psi \to \infty$. In this setting, leaders are not concerned with the value of a deal per se. Their primary goal is to strike a deal that helps them get reelected.

Retrospective Voters ($\rho = 1$)

Retrospective voters evaluate the incumbent by what he or she has delivered so far (Ferejohn 1986). Referring back to Figure 1, the median voter has their own indifference curve that induces an indifference policy $x_M = 1 - \left(\frac{s_0}{s_A}\right)^{\frac{\alpha_M}{1-\alpha_M}} (1 - x_0)$. Any deal such that $x_1 > x_M$ detracts from the median voter's welfare relative to the status quo and so hurts the incumbent's reelection prospects. In contrast, deals where $x_1 < x_M$ improve the incumbent's electoral prospects. To streamline the presentation, we focus, quite reasonably we believe, on the case where the median voter's own hawkishness lies between the hawk and dove parties, $\alpha_M \in (\alpha_R, \alpha_L)$.

Office-seeking leaders will only agree to deals that enhance their electoral prospects: $x_1 \leq x_M$. Although in policy terms L supports deals in the range $x_1 \in (x_M, x_L]$, he would reject any such offer because it harms his electoral prospects. Conversely, R would accept offers in the range (x_R, x_M), even though from a policy perspective she wants to reject them.

Retrospective voters and leaders with strong office-holding concerns effectively induce the ratification constraint articulated by Schelling (1960) and Putnam (1988). This constraint enhances a dove's bargaining leverage (Milner and Rosendorff 1997), whilst diminishing what an office-seeking hawk can extract.

Since voters are backward-looking, the type of incumbent (hawk or dove) does not affect his or her electoral prospects, it is simply the ability of the incumbent to have delivered greater welfare relative to

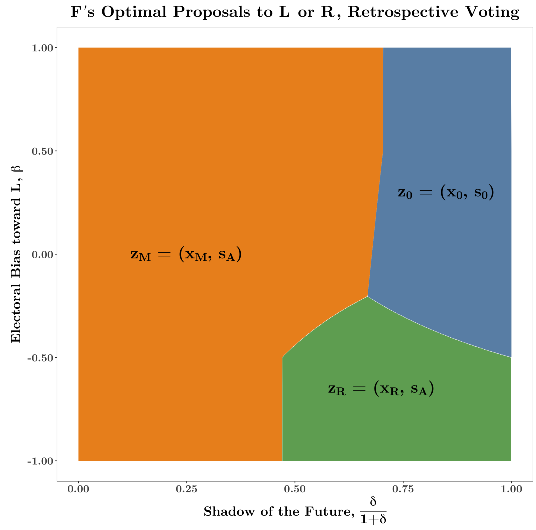


Figure 3: First Period Deals as a Function of Relative Importance of the Future (δ) and Electoral Bias toward L (β) with Retrospective Voters.

the status quo. In equilibrium as $\Psi \to \infty$ there are three possible first period equilibrium outcomes: z_0, z_R and $z_1 = (x_1, s_A)$ where $x_1 \to x_M$. In Figure 3, we label this last alternative z_M.

When the election is imminent (δ large), then F forgoes first period concessions and simply extracts the largest possible concession after the election (blue region in Figure 3). If the electoral bias favors R (small β) such that R is likely to win, then F offers z_R and such a deal persists through both rounds (green region).

Absent an imminent election or a large electoral bias in favor of R, F offers a deal that converges to the alliance deal that makes the median voter indifferent between an alliance-policy deal and no alliance, z_M (orange region). z_M is the largest policy concession that either leader will agree to. If F offers this deal then it obtains considerable policy concessions in the first round. However, should R win the election, the deal will be renegotiated to z_F.

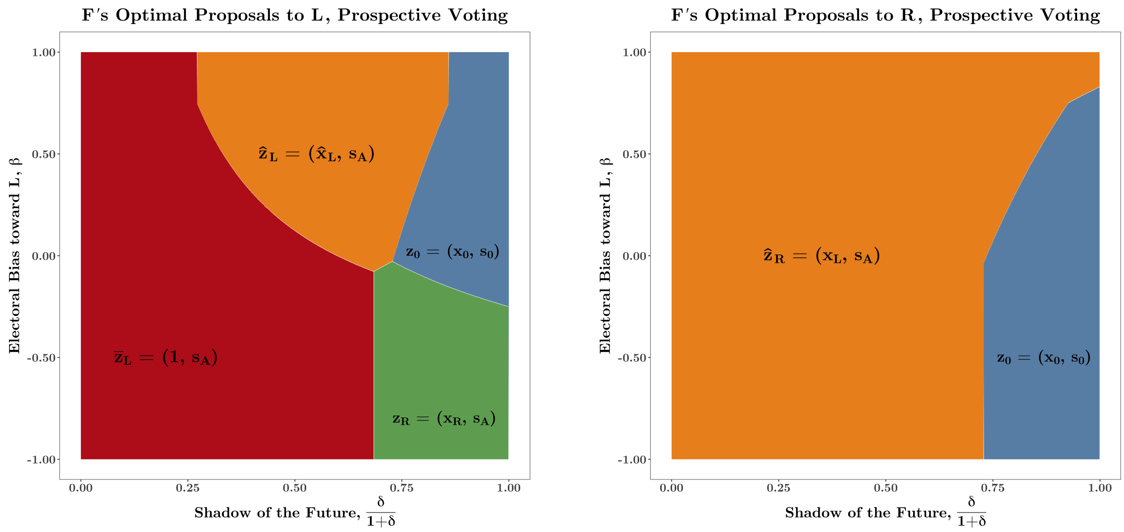
Prospective Voters ($\rho = 0$)

A prospective electorate selects a candidate by anticipating the downstream alliance-policy agreement. Voters consider who can deliver greater utility in second period negotiations and cast their votes in favor of that candidate. Being more hawkish, R has a post-electoral bargaining advantage relative to L, which makes her more attractive to voters. In pre-election bargaining R wants outcomes that enhance this bargaining advantage, while L seek outcomes that diminish it.

In a prospective setting, R is electorally favored relative to L if the first round deal is such that $x_1 \in (x_R, x_L]$ or if there is no deal, $z_1 = z_0$. In contrast, if $x_1 \in [x_F, x_R]$ or $x_1 > x_L$, both R and L deliver the same post-electoral deal (z_1 and z_F respectively). To enhance his electoral prospects L wants one of the latter scenarios, while R promotes her electoral chances through the former contingencies. F exploits L and R's electoral incentives.

Incumbent L**.** Figure 4 illustrates the first round deals within the prospective election setting. As the first panel shows, when L is the incumbent there are four possible outcomes. In common with earlier cases, when an election is imminent, F prefers to wait and offer z_0 (blue region) rather than tie its hands. Again, with similarities to previously considered settings, when an election is close and R is expected to win, F offers z_R, a deal that persists across both periods (green region). When an election is more distant, F exploits L's desire to make an agreement that reduces R's future bargaining advantage.

L is willing to agree to $x_1 = 1$, the largest possible policy concession. Given such a large concession, both leaders would renegotiate after the election ($z_2 = z_F$). By giving up so much that even a dove could walk away from the alliance after the election, L removes R's bargaining, and hence electoral, advantage. When the election is distant (small δ), F exploits L's willingness to agree to maximal concessions (red region).



(a) Incumbent L (b) Incumbent R

Figure 4: First Period Deals as a Function of Relative Importance of the Future (δ) and Electoral Bias toward L (β) with Prospective Voters.

Although F can extract maximal concessions from L, doing so leads to renegotiation with the outcome being z_F. If F is a little more patient and believes that L can win the election, then it prefers to propose a deal $z_1 = \widehat{z}_L$ that does not extract maximal concessions but will not be renegotiated if L wins (orange region). The logic behind this proposal is as follows. If no agreement is reached, then post-election deals are either z_R or z_L: this favors R electorally by $\sigma(V_M(x_R, s_A) - V_M(x_L, s_A))$. Note that any deal where $x_1 \in (x_R, x_L]$ benefits R by $\sigma(V_M(x_F, s_A) - V_M(x_1, s_A))$, as such deals would be maintained by L but renegotiated by R. The deal $\widehat{x}_L$ is such that $\sigma(V_M(x_F, s_A) - V_M(\widehat{x}_L, s_A)) = \sigma(V_M(x_R, s_A) - V_M(x_L, s_A))$, meaning that the electoral advantage is the same as not signing an agreement at all. If the election is moderately close and the electoral bias is in L's favor (large β), then F prefers to extract this smaller deal that would not be renegotiated if L (the likely winner) wins rather than extract the maximal concession.

Thus, with a prospective electorate, L is willing to make large concessions provided that they diminish R's post-electoral bargaining advantage. F exploits L's willingness to make large concessions given that the election is not too close.

Incumbent R. In the prospective electoral setting, the hawkish leader, R, refuses to make an agreement unless it increases her post-electoral bargaining advantage relative to L. The possible first round deals are z_0 and z_L.

If R rejects an agreement, then voters receive $\sigma(V_M(x_R, s_A) - V_M(x_L, s_A))$ more if she is elected. An office-seeking hawk will not agree to any deal that diminishes this electoral advantage. However, it is not the case that R rejects all agreements: if $x_1 \in (x_R, x_L]$, then R still retains her electoral advantage over L. F then proposes the largest concession that R would agree to, z_L. By making a large concession that R herself would renegotiate, but not so large that L could also credibly renegotiate, R increases her post-electoral advantage as she can deliver even more relative to L, $\sigma(V_M(x_F, s_A) - V_M(x_L, s_A))$.

The second panel of Figure 4 illustrates these deals as a function of the shadow of the election and electoral bias. When the election is close F prefers to wait, so the outcome is z_0 (blue region). When the election is more distant, F offer z_L which R accepts (orange region).

Summary: An office-seeking dove agrees to any deal such either both R and L would renegotiate after the election, or neither could renegotiate. Doing so dissipates R's electoral advantage. In contrast, R, an office-seeking hawk, only agrees to deals that she would renegotiate but that L could not. Doing so preserves her electoral advantage.

When domestic leaders are driven by office-holding concerns, they agree to almost any terms so long as it promotes their electoral odds. This willingness to give up policy to enhance electoral prospects enables F to obtain superior deals. If the election is reasonably distant, then F exploits both a hawk and dove's willingness to make deals to extract great immediate concessions, although at the risk of future renegotiations. In contrast, if an election is close, F simply waits in the hope of exploiting a dove after the election.

1. When an election is close, F waits to see who wins and maximally exploits the second period leader (z_0). If a dove is the incumbent, F may also choose to tie its hands by offering a deal that cannot

be renegotiated by either party (z_R), which it may be inclined to do if a hawk is likely to win the election.

2. When the election is further away, F exploits domestic leaders' electoral incentives. Doves will agree to any deal that does not differentiate themselves from hawks in post-election negotiations ($\overline{z}_L$). Hawks only agree to deals that enhance their electoral advantage (z_L). If F is a bit more patient and believes L will win the election, it may also offer a smaller concession today with the knowledge that it will not be renegotiated later ($\widehat{z}_L$).

B.5 Discussion

One common claim in the literature is that hawks have greater bargaining leverage than doves. Our model exhibits this feature when leaders are myopic. However, when elections and renegotiations are taken into account the predictions are more nuanced. Differences in bargaining leverage of hawks and doves drive many of the results, but they often do so off the equilibrium path as leaders on both sides factor in how agreements today affect future negotiations and electoral outcomes.

Table 3 summarizes many of the core predictions. Agreements reached today shape the bargaining leverage of different parties tomorrow and strategies of voters at the next election.

Several key themes run through the analysis. Nations F and D want to make agreements and delaying agreement is inefficient. However, concluding an agreement ties F's hands. If an agreement is relatively generous toward D ($x_1 \leq x_R$) then the agreement persists and F forgoes the ability to exploit L's dovishness in the future. In contrast, if F obtains large immediate concessions, then it risks unfavorable renegotiations should a hawk come to power. When an election is imminent, F avoids either of these eventualities and simply waits until after the election to conclude an agreement.

When the elections are further off then predictions depend upon the incumbent, the electoral setting, and the likelihood of each party winning. Today's deal determines future agreements. When a hawk is likely to be in power in the future, then F offers an agreement that both hawks and doves would be happy with (z_R). In contrast, if a dove is likely to be in power in the future, or the election is far off, then F maximally exploits the current leader.

Electoral concerns moderate our predictions. In a retrospective electoral context, voters reward the incumbent for improving the status quo. To ingratiate themselves with the voters, doves reject deals that the voters dislike ($x_1 > x_M$), even if they would agree to such deals on policy terms. And office-seeking hawks are willing to make additional concessions over what they support on policy grounds in order to deliver benefits to the voters, for which they hope to be rewarded. When leaders are primarily driven by office-holding desires, and when elections are far off, the predictions resemble those espoused by Schelling

(1960) and Putnam (1988) in which the voters effectively function as a ratification constraint.

	Distant Election (δ small)	Imminent Election (δ large)
	Myopic Leaders ($\delta = 0$)	
Incumbent L	z_L	z_L
Incumbent R	z_R	z_R
	Exogenous Election ($\sigma = 0$)	
Incumbent L	$\overline{z}_L$ if R expected winner z_L if L expected winner	z_R if R expected winner z_0 if L expected winner
Incumbent R	z_R if R expected winner $\widehat{z}_R$ if L expected winner	z_R if R expected winner z_0 if L expected winner
	Retrospective Voting ($\Psi \to \infty$)	
Incumbent L	z_M	z_R if R expected winner z_0 if L expected winner
Incumbent R	z_M	z_R if R expected winner z_0 if L expected winner
	Prospective Voting ($\Psi \to \infty$)	
Incumbent L	$\overline{z}_L = (1, s_A)$	$\widehat{z}_L$ if L expected winner z_R if R expected winner z_0 if very high δ
Incumbent R	z_L	z_0

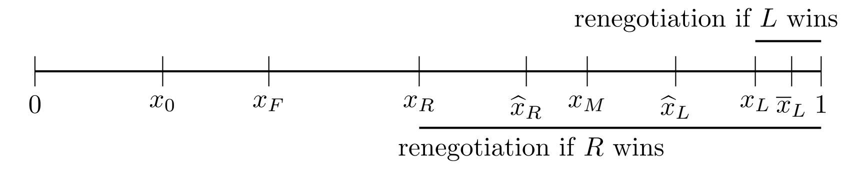


Table 3: Comparison of Policies Under Different Electoral Scenarios

When the voters are prospective, hawks seek to differentiate themselves from doves by rejecting any deal that undercuts their innate electoral advantage. Doves, by contrast, will grant large concessions provided that they will result in identical post-electoral bargaining outcomes. F readily capitalizes on these electoral incentives and can extract large immediate policy concessions from hawks and doves. The desires of incumbents to maximize their electoral prospects can result in leaders undermining their immediate policy successes. Paradoxically, this can even lead to doves cutting better deals than hawks (compare $\widehat{z}_L$ to z_L in Figure 4).

The prospect of future renegotiations drives deal-making today. In many bargaining models there is both an inherent first-mover advantage as well as a prediction that instantaneous, efficient outcomes are negotiated (Rubinstein 1982). In our model, it is a perfectly rational decision to forgo cooperation today with the expectation that one can conclude more favorable terms tomorrow or that rejecting an agreement could enhance electability. While post-election outcomes are efficient on the equilibrium path, these bargaining surpluses need not be reached immediately. In other words, the second period leader's credible threat to walk away from negotiations influences how F proposes (or does not propose) concessions in the first period, how first period leaders choose to accept or reject those deals in relation to their electoral fortunes, and how post-electoral negotiations are defined.

B.6 Proofs

B.6.1 Second Period Agreements

Lemma 1 *Consider second period negotiations given a first period outcome z_1:*

1. *If there is no alliance, $z_1 = z_0$, then $z_2 = z_R$ if R is the second period leader and $z_2 = z_L$ if L is the second period leader.*
2. *If there is an alliance, $z_1 = (x_1, s_A)$ and $x_1 < x_F$, then $z_2 = z_F$.*
3. *If there is an alliance, $z_1 = (x_1, s_A)$, and $x_1 \in [x_F, x_R]$, then $z_2 = z_1$, i.e. alliance remains unchanged.*
4. *If there is an alliance, $z_1 = (x_1, s_A)$, and $x_1 \in (x_R, x_L]$, then $z_2 = z_1$ if L is the second period leader and $z_2 = z_F$ if R is the second period leader.*
5. *If there is an alliance, $z_1 = (x_1, s_A)$, and $x_1 > x_L$, then $z_2 = z_F$.*

Proof of Lemma 1: If there is no alliance, then L will accept agreement iff $x_2 \leq x_L$ and R will accept agreement iff $x_2 \leq x_R$. To maximize its payoff, F proposes the largest policy that L or R will accept. Hence item 1.

Next note that if $x_1 \in [x_F, x_i]$ for $i = L, R$ then both sides prefer that the deal persists rather than exit (which results in z_0). Since such agreements are Pareto efficient, i cannot make payoff improving renegotiation proposal that F will accept. Such deals persist.

If either $x_1 < x_F$ or $x_1 > x_i$ then either F or i will exit the agreement unless it is renegotiated. Since $z_1 \neq z_0$, i has proposal power. F will accept $x_2 \geq x_F$ rather than return to z_0. i offers its most preferred acceptable policy, i.e. x_F. ■

Lemma 2 *There is never an alliance (x, s_A) in which $x < x_F$.*

Proof of Lemma 2: Consider the second period: such an alliance is worse for F than the status quo: $V_F(x, s_A) < V_F(z_0)$ if $x < x_F$. Hence in the second period F would never propose such an alliance and would exit any agreement that would result in such an alliance.

Next consider the first period. If such an alliance was the first period agreement, then by Lemma 1, the second period outcome is x_F. F can always engineer the status quo (z_0) in both periods and since $V_F(x_1, s_A) + \delta V_F(z_F) < (1+\delta)V_F(z_0)$, there are never agreements such that $x_1 < x_F$. Note this analysis does not depend on which party is elected and so holds for all electoral variations. ■

It follows directly from Lemma 2 that either $z_1 = z_0$ or $z_1 = (x_1, s_A)$ where $x_1 \geq x_F$.

Via Lemma 1, for each first period outcome and (second period) leader there is a unique second period outcome. We structure the analysis of first period outcomes as follows: we consider each incumbent leader in turn and divide the policy space into three regions: $x \in [x_F, x_R]$, $x \in (x_R, x_L]$ and $x \in (x_L, 1]$. Through a series of lemmas we characterize the set of policies within each region that i will accept (rather than z_0) and find the largest policy in each region that F can obtain. For instance, if L is the incumbent, lemmas 3, 4 and 5 characterize $\widetilde{x}_L \in [x_F, x_R]$, $\widehat{x}_L \in (x_R, x_L]$ and $\overline{x}_L \in (x_L, 1]$, respectively, that define the largest attainable deals available to F. Proposition 1 states that F picks its most desirable first period outcome: z_0, $\widetilde{z}_L$, $\widehat{z}_L$ or $\overline{z}_L$. Lemmas 6, 7 and 8 and proposition 2 provide analogous analyzes when R is the incumbent.

B.6.2 Dovish Incumbent

We annotate equations using underbraces to indicate how bargains affect the electability of L.

If L is the incumbent and there is no deal in the first period, then L's payoff is

$$U_L(z_0, L) = \begin{cases} V_L(z_0) + \delta V_L(z_R) + \delta G[\beta]\,(\Psi + V_L(z_L) - V_L(z_R)) & \text{if} \quad \rho = 1 \\ V_L(z_0) + \delta V_L(z_R) + \delta G[\beta + \sigma(\underbrace{V_M(z_L) - V_M(z_R)}_{-})]\,(\Psi + V_L(z_L) - V_L(z_R)) & \text{if} \quad \rho = 0 \end{cases} \tag{2}$$

The negative sign under $V_M(z_L) - V_M(z_R)$ indicates that under prospective voting, L is electorally harmed by no agreement. After the election the deal R would strike is z_R, which is more advantageous to the median voter than z_L. F's payoff from no agreement under L's incumbency is

$$U_F(z_0, L) = \begin{cases} V_F(z_0) + \delta V_F(z_R) + \delta G[\beta]\,(V_F(z_L) - V_F(z_R)) & \text{if} \quad \rho = 1 \\ V_F(z_0) + \delta V_F(z_R) + \delta G[\beta + \sigma(V_M(z_L) - V_M(z_R))]\,(V_F(z_L) - V_F(z_R)) & \text{if} \quad \rho = 0 \end{cases} \tag{3}$$

Agreements with L where $x_1 \in [x_F, x_R]$

Lemma 3 *Suppose L is first period leader. If $z_1 = (x_1, s_A)$ such that $x_1 \in [x_F, x_R]$ then $z_1 = z_R = (x_R, s_A)$. F's payoff from such a deal is*

$$U_F(z_1) = V_F(z_R)(1 + \delta) \tag{4}$$

Proof of Lemma 3: Via Lemma 1, any such agreement will persist for both rounds. If L accepts such an agreement z_1, then his payoff is

$$U_L(z_1, L) = \begin{cases} V_L(z_1)(1+\delta) + \delta G[\beta + \sigma(\underbrace{V_M(z_1) - V_M(z_0)}_{+})]\Psi & \text{if} \quad \rho = 1 \\ V_L(z_1)(1+\delta) + \delta G[\beta]\Psi & \text{if} \quad \rho = 0 \end{cases} \tag{5}$$

Since $V_M(z_1) - V_M(z_0) > 0$ and $V_M(z_L) - V_M(z_R) < 0$ such an agreement enhances L's reelection chances relative to z_0 under both retrospective and prospective voting. Further, any such deal is better in policy terms than what L can obtain from z_0. F's payoff is increasing in x_1 in this range and so F would offer $z_1 = z_R$. ∎

Agreements with L where $x_1 \in (x_R, x_L]$

For $z_1 = (x_1, s_A)$ such that $x_1 \in (x_R, x_L]$, L's payoff for such an agreement is

$$U_L(z_1, L) = \begin{cases} V_L(z_1) + \delta V_L(z_F) + \delta G[\beta + \sigma(\underbrace{V_M(z_1) - V_M(z_0)}_{+/-})](\Psi + V_L(z_1) - V_L(z_F)) & \text{if} \quad \rho = 1 \\ V_L(z_1) + \delta V_L(z_F) + \delta G[\beta + \sigma(\underbrace{V_M(z_1) - V_M(z_F)}_{-})](\Psi + V_L(z_1) - V_L(z_F)) & \text{if} \quad \rho = 0 \end{cases} \tag{6}$$

Define $\widehat{z}_L = (\widehat{x}_L, s_A)$ as deal with the largest $x_1 \in (x_R, x_L]$ such that equation 6 is at least as big as equation 2, if any such deal exists. This deal defines the largest policy concession that L will accept in the range $x_1 \in (x_R, x_L]$.

Lemma 4 *Suppose L is first period leader. If $z_1 = (x_1, s_A)$ such that $x_1 \in (x_R, x_L]$, then $z_1 = \widehat{z}_L = (\widehat{x}_L, s_A)$.*

Proof of Lemma 4: Via Lemma 1, if there is such a first round deal, then $z_2 = z_F$ if R elected and $z_2 = z_1$ if L is elected.

Given assumption 1, that office holding is important ($\Psi > 1$), then $U_L(z_1, L)$ (equation 6) is strictly decreasing in x_1. F picks the policy that maximizes its welfare in the range $x_1 \in (x_R, \widehat{x}_L]$.

$$U_F(z_1, L) = \begin{cases} V_F(z_1) + \delta V_F(z_F) + \delta G[\beta + \sigma(V_M(z_1) - V_M(z_0))](V_F(z_1) - V_F(z_F)) & \text{if} \quad \rho = 1 \\ V_F(z_1) + \delta V_F(z_F) + \delta G[\beta + \sigma(V_M(z_1) - V_M(z_F))](V_F(z_1) - V_F(z_F)) & \text{if} \quad \rho = 0 \end{cases} \tag{7}$$

If $\rho = 1$ then

$$\begin{aligned} \frac{dU_F(z_1, L)}{dx_1} &= \frac{dV_F(z_1)}{dx_1}(1 + \delta G[\beta + \sigma(V_M(z_1) - V_M(z_0))]) \\ &+ \delta\sigma \frac{dV_M(z_1)}{dx_1} g[\beta + \sigma(V_M(z_1) - V_M(z_0))](V_F(z_1) - V_F(z_F)) \end{aligned} \tag{8}$$

The first term of which is positive and the second term is negative. However, given assumption 2 (that security policy is not too salient), the first term dominates the second and so F's payoff is increasing in x_1 (there is a close analogous condition if $\rho = 0$). Hence if there is a settlement in this range, then F offers $\widehat{x}_L$, that L would accept. ■

Note that absent assumption 2, F might prefer to offer an intermediate value of $x_1 \in (x_R, \widehat{x}_L)$ that satisfies a first condition defined by equation 8 to increase the chance of facing leader L in the second period.

Agreements with L where $x_1 \in (x_L, 1]$

For $z_1 = (x_1, s_A)$ such that $x_1 \in (x_L, 1]$, L's payoff for such an agreement is

$$U_L(z_1, L) = \begin{cases} V_L(z_1) + \delta V_L(z_F) + \delta G[\beta + \sigma\underbrace{(V_M(z_1) - V_M(z_0))}_{-}]\Psi & \text{if} \quad \rho = 1 \\ V_L(z_1) + \delta V_L(z_F) + \delta G[\beta]\Psi & \text{if} \quad \rho = 0 \end{cases} \tag{9}$$

Define $\overline{z}_L = (\overline{x}_L, s_A)$ as deal where $\overline{x}_L$ is the largest $x_1 \in (x_L, 1]$ such that equation 9 is at least as great as equation 2, when such a deal exists. This deal is largest deal that L accepts above x_L. Note that in the first period L gets a deal that he myopically wants to reject, but in the second period he will receive his most preferred implementable deal (z_F).

Lemma 5 *Suppose L is first period leader. If $z_1 = (x_1, s_A)$ such that $x_1 > x_L$, then $z_1 = \overline{z}_L = (\overline{x}_L, s_A)$ and F's payoff is*

$$U_F(z_1, L) = V_F(z_1) + \delta V_F(z_F) \tag{10}$$

Proof of Lemma 5: If $x_1 > x_L$ then by Lemma 1 the second period outcome is z_F. Hence L's payoff is given by equation 9. Increases in x_1 reduce L's payoff directly and in the retrospective case harm L's electoral prospects. Hence L's payoff is decreasing in x_1. Define $\overline{x}_L$ as the largest $x_1 \in (x_L, 1]$ such that equation 9 is at least as large as equation 2. This is largest deal that L is willing to accept above x_L. F's payoff (equation 10) is strictly increasing in x_1, so F would ask for the largest deal that L would accept. Hence if $x_1 > x_L$, then $z_1 = \overline{z}_L = (\overline{x}_L, s_A)$. ■

The lemmas above identify what deals would look like is they occurred in certain areas of the policy space. The following proposition simply states that the first period bargain must be one of these deals.

Proposition 1 *If L is incumbent, then $z_1 \in \{z_0,\ z_R,\ \widehat{z}_L = (\widehat{x}_L, s_A),\ \overline{z}_L = (\overline{x}_L, s_A)\}$. With the deal being that associated with the largest payoff for F, given by equations 3, 4, 7 and 10, respectively.*

B.6.3 Hawkish Incumbent

As above, we proceed with a series of lemmas. If there is no deal in the first period, then R's payoff is

$$U_R(z_0, R) = \begin{cases} V_R(z_0) + \delta(\Psi + V_R(z_R) + G[\beta]\,(-\Psi + V_R(z_L) - V_R(z_R))) & \text{if} \quad \rho = 1 \\ V_R(z_0) + \delta(\Psi + V_R(z_R) + G[\beta + \sigma(\underbrace{V_M(z_L) - V_M(z_R)}_{-})]\,(-\Psi + V_R(z_L) - V_R(z_R))) & \text{if} \quad \rho = 0 \end{cases} \tag{11}$$

Under prospective voting, R can deliver a superior outcome for M, which harms L's electoral prospects. Under no deal F's payoff is

$$U_F(z_0, R) = \begin{cases} V_F(z_0) + \delta V_F(z_R) + \delta G[\beta](V_F(z_L) - V_F(z_R)) & \text{if} \quad \rho = 1 \\ V_F(z_0) + \delta V_F(z_R) + \delta G[\beta + \sigma(V_M(z_L) - V_M(z_R))](V_F(z_L) - V_F(z_R)) & \text{if} \quad \rho = 0 \end{cases} \tag{12}$$

Agreements with R where $x_1 \in [x_F, x_R]$

Consider $z_1 = (x_1, s_A)$ such that $x_1 \in [x_F, x_R]$. R's payoff from such a deal is

$$U_R(z_1, R) = \begin{cases} V_R(z_1)(1+\delta) + \Psi(1 - G[\beta + \sigma(\underbrace{V_M(z_0) - V_M(z_1)}_{-})]) & \text{if} \quad \rho = 1 \\ V_R(z_1)(1+\delta) + \Psi(1 - G[\beta]) & \text{if} \quad \rho = 0 \end{cases} \tag{13}$$

Under retrospective voting, such a deal enhances R's electoral prospects; however, under prospective voting there is no electoral advantage. Define $\widetilde{x}_R$ as the largest $x_1 \in [x_F, x_R]$ such that equation 13 at least as big as equation 11.

Lemma 6 *Suppose R is first period leader. If $z_1 = (x_1, s_A)$ such that $x_1 \in [x_F, x_R]$, then $z_1 = \widetilde{z}_R = (\widetilde{x}_R, s_A)$. F's payoff from such a deal is*

$$U_F(z_1, R) = V_F(z_1)(1+\delta) \tag{14}$$

.

Proof of Lemma 6: Via Lemma 1, any such agreement will persist for both rounds. R's payoff is decreasing in $x_1 \in [x_F, x_R]$ so $\widetilde{x}_R$ defines the largest deal that R will accept. F's payoff is increasing $x_1 \in [x_F, x_R]$. So F proposes the maximum that R will accept, $\widetilde{z}_R$. If $\rho = 1$, then $\widetilde{x}_R = x_R$. ■

Agreements with R where $x_1 \in (x_R, x_L]$

For $z_1 = (x_1, s_A)$ such that $x_1 \in (x_R, x_L]$, R's payoff for such an agreement is

$$U_R(z_1,R) = \begin{cases} V_R(z_1) + \delta V_R(z_F) + \delta\Psi + \delta G[\beta + \sigma(\underbrace{V_M(z_0) - V_M(z_1)}_{+/-})](-\Psi + V_R(z_1) - V_R(z_F)) & \text{if} \quad \rho = 1 \\ V_R(z_1) + \delta V_R(z_F) + \delta\Psi + \delta G[\beta + \sigma(\underbrace{V_M(z_1) - V_M(z_F)}_{-})](-\Psi + V_F(z_1) - V_F(z_F)) & \text{if} \quad \rho = 0 \end{cases} \tag{15}$$

Define $\widehat{z}_R = (\widehat{x}_R, s_A)$ as deal with the largest $x_1 \in (x_R, x_L]$ such that equation 15 is at least as big as equation 11.

Lemma 7 *Suppose R is first period leader. If $z_1 = (x_1, s_A)$ such that $x_1 \in (x_R, x_L]$, then $z_1 = \widehat{z}_R = (\widehat{x}_R, s_A)$.*

Proof of Lemma 7: Via Lemma 1, if there is such a first round deal then $z_2 = z_F$ if R elected and $z_2 = z_1$ if L elected.

$$U_F(z_1,R) = \begin{cases} V_F(z_1) + \delta V_F(z_F) + \delta G[\beta + \sigma(V_M(z_0) - V_M(z_1))](V_F(z_1) - V_F(z_F)) & \text{if} \quad \rho = 1 \\ V_F(z_1) + \delta V_F(z_F) + \delta G[\beta + \sigma(V_M(z_1) - V_M(z_F))](V_F(z_1) - V_F(z_F)) & \text{if} \quad \rho = 0 \end{cases} \tag{16}$$

If $\rho = 1$ then

$$\begin{aligned} \frac{dU_F(z_1,R)}{dx_1} &= \frac{dV_F(z_1)}{dx_1}(1 + \delta G[\beta + \sigma(V_M(z_0) - V_M(z_1))]) \\ &- \delta\sigma\frac{dV_M(z_1)}{dx_1} g[\beta + \sigma(V_M(z_0) - V_M(z_1))](V_F(z_1) - V_F(z_F)) \end{aligned} \tag{17}$$

The first term is positive and, given assumption 2, dominates the second term. If $\rho = 0$, then all terms are positive. Therefore, F proposes the largest deal that R will accept, $\widehat{x}_R$. ■

Corollary 1 *Under prospective elections ($\rho = 0$) with high returns to office-holding ($\Psi \to \infty$), $\widehat{z}_R = z_L$.*

Proof of Corollary 1: We characterize $\widehat{z}_R = (\widehat{x}_R, s_A)$ under different electoral assumptions. If voters are retrospective, then $\frac{dU_R(z_1,R)}{dx_1} < 0$. However, if $\rho = 0$,

$$\begin{aligned} \frac{dU_R(z_1,R)}{dx_1} &= \frac{dV_R(z_1)}{dx_1}(1 + \delta G[\beta + \sigma(V_M(z_0) - V_M(z_1))]) \\ &+ \sigma\frac{dV_M(z_1)}{dx_1}\delta g[\beta + \sigma(V_M(z_1) - V_M(z_F))](-\Psi + V_R(z_1) - V_R(z_F)) \end{aligned} \tag{18}$$

The first term is negative, however the second term is positive. As Ψ becomes large the latter term is dominant indicating that R prefers 'worse' deals because such deals harm L's electoral prospects. Hence, $\widehat{z}_R = z_L$. ■

Agreements with R where $x_1 \in (x_L, 1]$

For $z_1 = (x_1, s_A)$ such that $x_1 \in (x_L, 1]$, R's payoff for such an agreement is

$$U_R(z_1, R) = \begin{cases} V_R(z_1) + \delta V_R(z_F) + \delta\Psi - \delta\Psi G[\beta + \sigma(\underbrace{V_M(z_0) - V_M(z_1)}_{+})] & \text{if} \quad \rho = 1 \\ V_R(z_1) + \delta V_R(z_F) + \delta\Psi - \delta\Psi G[\beta] & \text{if} \quad \rho = 0 \end{cases} \tag{19}$$

Define $\overline{z}_R = (\overline{x}_R, s_A)$ as deal with the largest $x_1 \in (x_L, 1]$ such that equation 19 is at least as big as equation 11.

Lemma 8 *Suppose R is first period leader. If $z_1 = (x_1, s_A)$, such that $x_1 \in (x_R, x_L]$, then, $z_1 = \overline{z}_R = (\overline{x}_R, s_A)$.*

Proof of Lemma 8: Via Lemma 1, if there is such a first round deal, then $z_2 = z_F$. Clearly, for such a deal R's payoff is decreasing in x_1. F's payoff for a deal in this range is

$$U_F(z_1, F) = V_F(z_1) + \delta V_F(z_F), \tag{20}$$

which is increasing in x_1. If there is such an offer, then F proposes the largest deal that R accepts. ■
The above lemmas identify the deals in different regions of the policy space. The following proposition simply states that the first period outcome must be one of these deals.

Proposition 2 *If R is incumbent, then $z_1 \in \{z_0,\ \widetilde{z}_R = (\widetilde{x}_R, s_A),\ \widehat{z}_R = (\widehat{x}_R, s_A),\ \overline{z}_R = (\overline{x}_R, s_A)\}$. With the deal being that associated with the largest payoff for F, given by equations 12, 14, 16 and 20, respectively.*

B.6.4 Limiting Cases

Myopic Leaders

Corollary 2 *If leaders are myopic such that $\delta = 0$, then $z_1 = z_L$ if L is the incumbent and $z_1 = z_R$ if R is the incumbent.*

This corollary is simply the trivial case of the single shot game.

Exogenous Elections

If elections are insensitive to the security alliance ($\sigma = 0$), then L is elected with probability $p = G[\beta]$ under all contingencies. Bargaining is unaffected by office concerns. Given the above lemmas we characterize $\widetilde{x}_L = x_R$, $\widehat{x}_L = x_L$, $\overline{x}_L$, $\widetilde{x}_R$, $\widehat{x}_R$ and $\overline{x}_R$.

When L is incumbent, $\widehat{z}_L$ is characterized by

$$V_L(\widehat{z}_L)(1+\delta p) + \delta(1-p)V_L(z_F) = V_L(z_L)(1+\delta p) + \delta(1-p)V_L(z_R)$$

(in which the RHS is L's payoff from z_0) which for an interior solution $\widehat{x}_L \in (x_R, x_L)$ implies

$$\begin{aligned}
\widehat{x}_L &= 1 - \left(\frac{\delta(1-p)\left((1-x_R)^{1-\alpha_L} - (1-x_F)^{1-\alpha_L}\right)}{\delta p + 1} + (1-x_L)^{1-\alpha_L} \right)^{\frac{1}{1-\alpha_L}} \\
&= 1 - \frac{515.611(-0.0102911\delta + 0.0731397\delta p + 0.0628486)^3}{(\delta p + 1)^3} \qquad (21)
\end{aligned}$$

$\overline{z}_L$ is characterized by

$$V_L(\overline{z}_L) + \delta V_L(z_F) = V_L(z_L)(1+\delta p) + \delta(1-p)V_L(z_R)$$

which for an interior solution $\overline{z}_L \in (x_L, 1)$ implies

$$\begin{aligned}
\overline{x}_L &= 1 - \left(-\delta\,(1-x_F)^{1-\alpha_L} + \delta(1-p)\,(1-x_R)^{1-\alpha_L} + (\delta p + 1)\,(1-x_L)^{1-\alpha_L}\right)^{\frac{1}{1-\alpha_L}} \\
&= (0.0825219\delta + 0.292876\delta p - 0.503968)^3 \qquad (22)
\end{aligned}$$

Likewise if R is incumbent:

$\widetilde{z}_R = z_R$ since

$$V_R(z_R)(1+\delta) > V_R(z_R)(1+\delta(1-p)) + \delta p V_R(z_L)$$

$\widehat{z}_R$ is characterized by

$$V_R(\widehat{z}_R)(1+\delta p) + \delta(1-p)V_R(z_F) = V_R(z_R)(1+\delta(1-p)) + \delta p V_R(z_L)$$

which for an interior solution $\widehat{z}_R \in (x_R, x_L)$ implies

$$\begin{aligned}
\widehat{x}_R &= 1 - \left(-\frac{\delta(1-p)\,(1-x_F)^{1-\alpha_R}}{\delta p + 1} + \frac{\delta p\,(1-x_L)^{1-\alpha_R}}{\delta p + 1} + \frac{(\delta(1-p)+1)\,(1-x_R)^{1-\alpha_R}}{\delta p + 1} \right)^{\frac{1}{1-\alpha_R}} \\
&= 1 - (-0.138324\delta - 0.380976\delta p + 0.63496)^{3/2} \qquad (23)
\end{aligned}$$

$\overline{z}_R$ is characterized by

$$V_R(\overline{z}_R) + \delta V_R(z_F) = V_R(z_R)(1 + \delta(1-p)) + \delta p V_R(z_L)$$

which for an interior solution $\overline{z}_R \in (x_L, 1)$ implies

$$\begin{aligned} \overline{x}_R &= 1 - \left(-\delta\left(1-x_F\right)^{1-\alpha_R} + \delta p\left(1-x_L\right)^{1-\alpha_R} + \left(\delta(1-p)+1\right)\left(1-x_R\right)^{1-\alpha_R}\right)^{\frac{1}{1-\alpha_R}} \\ &= 1 - (-0.138324\delta + 0.253984\delta p - 0.63496\delta p + 0.63496)^{3/2} \end{aligned} \quad (24)$$

Retrospective Voting

We focus on office-seeking motivations: $\rho = 1$ and $\Psi \to \infty$.
If L is incumbent, then $\widetilde{x}_L = x_R$, $\widehat{x}_L \to x_M \in (x_R, x_L)$ and L never agrees to any z_1 such that $x_1 > x_M$.
If R is incumbent, then $\widetilde{x}_R = x_R$ and $\widehat{x}_R \to x_M \in (x_R, x_L)$. R never agrees to any z_1 such that $x_1 > x_M$.
Hence, F can obtain either z_0, z_R or $z_M \to (x_M, s_A)$ with payoffs

$$\begin{aligned} U_F(z_0) &= V_F(z_0) + \delta G[\beta] V_F(z_L) + \delta(1 - G[\beta]) V_F(z_R) \\ U_F(z_R) &= V_F(z_R)(1+\delta) \\ U_F(z_M) &\to V_F(z_M)(1 + \delta G[\beta]) + \delta(1 - G[\beta]) V_F(z_F) \end{aligned}$$

Prospective Voting

We consider office holding as the dominant motivation: $\rho = 0$ and $\Psi \to \infty$. If L is incumbent, then $\widehat{z}_L$ is characterized by

$$V_M(\widehat{z}_L) - V_M(z_F) = V_M(z_L) - V_M(z_R)$$

That is to say in M's utility terms the difference between $\widehat{z}_L$ and z_F is the same as the difference between z_L and z_R. Therefore

$$\begin{aligned} \widehat{x}_L &= 1 - \left(\left(1-x_F\right)^{1-\alpha_M} + \left(1-x_L\right)^{1-\alpha_M} - \left(1-x_R\right)^{1-\alpha_M}\right)^{\frac{1}{1-\alpha_M}} \\ &= .7881 \end{aligned}$$

L will agree to any policy above x_L as it diminishes R electoral advantage: $\overline{z}_L = (1, s_A)$.

If R is incumbent, then R rejects all deals in $[x_F, x_R]$ and in $(x_L, 1]$ because any such deal diminishes electoral advantage because both L and R deliver the same policy in period 2. In the range (x_R, x_L)

increases in x_1 diminish the policy value of deals but enhance R's election prospects (Corollary 1). Thus, R will agree to $\widehat{x}_R = x_L$. Hence if R is incumbent, then $z_1 \in \{z_0, z_L\}$.

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