

Appendix: Shaming Paris: A Political Economy of Climate

Commitments

Justin Melnick and Alastair Smith

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Uniqueness of Optimal Mitigation Efforts

As eluded to in the main text, there can be multiple local maxima that satisfy the FOC characterized by equation 1. The government's payoff, $u_g(a_g, A; \lambda_g)$, is composed of two single peaked functions. In terms of policy, $a_g - \frac{a_g^2 \lambda_g}{2}$ is a concave function with a maximum at $a_g = \frac{1}{\lambda_g}$ that contributes the terms $1 - \lambda_g a_g$ to the FOC. The probability of being shamed is decreasing in a_g , and contributes the $\sigma_g \sqrt{\beta} \phi(\sqrt{\beta}(y - a_g))$ term to the FOC.

$$foc = \frac{du_g(a_g, A; \lambda_g)}{da_g} = 1 - \lambda_g a_g^* + \sigma_g \sqrt{\beta} \phi(\sqrt{\beta}(y - a_g^*))$$

and the second-order condition (SOC)

$$soc = \frac{d^2 u_g(a_g, A; \lambda_g)}{da_g^2} = -\lambda_g + \sigma_g \beta \sqrt{\beta}(y - a_g^*) \phi(\sqrt{\beta}(y - a_g^*)).$$

When signals are imprecise the government's payoff is globally concave and so there is only a single solution to $foc = 0$, as formally stated in the following lemma.

Lemma A.1 *If $\beta < \frac{\sqrt{2e\pi}\lambda_g}{\sigma_g}$ (or equivalently $\sigma_g < \frac{\sqrt{2e\pi}\lambda_g}{\beta}$), then the government's payoff $u_g(a_g, A; \lambda_g)$ is globally concave for any y , there is a unique solution to $foc = 0$ and $a_g^* \geq \frac{1}{\lambda_g}$.*

Proof of Lemma A.1: The second order condition is $soc = -\lambda_g + \sigma_g \beta \sqrt{\beta}(y - a_g) \phi(\sqrt{\beta}(y - a_g))$, which has a maximum of $\frac{\beta \sigma_g}{\sqrt{2e\pi}} - \lambda_g$ at $y - a_g = \frac{1}{\sqrt{\beta}}$. Hence if $\beta < \frac{\sqrt{2e\pi}\lambda_g}{\sigma_g}$ then soc is always negative and the government's optimization is globally concave and foc is decreasing in a_g . At $a_g = \frac{1}{\lambda_g}$, $foc \geq 0$ and as $a_g \rightarrow \infty$, $foc \rightarrow -\infty$, therefore there is a unique $a_g^* \geq \frac{1}{\lambda_g}$ such that $foc = 0$. ■

If signals are more precise then the government's utility function, $u_g(a_g, A; \lambda_g)$, is potentially two peaked with a peak around $a_g = \frac{1}{\lambda_g}$ and another peak around $a_g = y$. If y is relatively close to $\frac{1}{\lambda_g}$, then these two peaks coincide resulting in the aggregate $u_g(\cdot)$ being single peaked. In contrast if y is relatively large compared to $\frac{1}{\lambda_g}$, then $u_g(a_g, A; \lambda_g)$ is two peaked and there are two local maxima that satisfy the $foc = 0$ (and $soc < 0$). Further since $u_g(\cdot)$ is continuous, if there are two local maxima, then there must also be a local minimum between them that satisfies $foc = 0$ and $soc > 0$. The following lemma exploits this graphical exposition of the shape of $u_g(\cdot)$.

The first two conditions show that when y is relatively extreme (less than $\frac{1}{\lambda_g}$ or greater than $\frac{1 + \sqrt{2\lambda_g \sigma_g}}{\lambda_g}$), then, with precise signals, the government's effort is close to $\frac{1}{\lambda_g}$. The third condition exploits the fact that if there are two local maxima that satisfy $foc = 0$, then there must also be a local minimum between them. If signals are imprecise, then no such minimum can exist and therefore there is a unique local maximum. In

contrast, if signal are precise, then two local maxima that satisfy $foc = 0$ can exist and therefore a_g^* can be discontinuous in y .

Lemma A.2 1. If $y \leq \frac{1}{\lambda_g}$, then $a_g^* \geq \frac{1}{\lambda_g}$.

2. If $y \geq \frac{1+\sqrt{2\lambda_g\sigma_g}}{\lambda_g}$, then $a_g^* \in [1/\lambda_g, y)$.

3. $\beta < \frac{4\lambda_g^2}{(\lambda_g y - 1)^2}$ is sufficient to ensure there is a unique local maximum that satisfies $foc = 0$ and a_g^* is continuous in y . If $\beta > \frac{4\lambda_g^2}{(\lambda_g y - 1)^2}$ then there can be two maxima that satisfy $foc = 0$ and a_g^* can be discontinuous in y .

4. As $\beta \rightarrow \infty$, $a_g^* \rightarrow \max\{y, \frac{1}{\lambda_g}\}$ if $y < \frac{\sqrt{2}\sqrt{\lambda_g}\sqrt{\sigma_g}+1}{\lambda_g}$; and $a_g^* \rightarrow \frac{1}{\lambda_g}$ if $y > \frac{\sqrt{2}\sqrt{\lambda_g}\sqrt{\sigma_g}+1}{\lambda_g}$.

Proof of Lemma A.2: For part 1, if $y \leq \frac{1}{\lambda_g}$, then for $a_g < 1/\lambda_g$ the government's payoff is strictly increasing in a_g . For $a_g = 1/\lambda_g$, the $foc \geq 0$ and for all $a_g > 1/\lambda_g$, $soc < 0$, so foc is strictly decreasing in a_g for all $a_g \geq 1/\lambda_g$ and therefore the foc can only cross zero once.

For part 2 consider the following limiting cases. The government can always play $a_g = \frac{1}{\lambda_g}$ and get a payoff at least as big as $\frac{1}{2\lambda_g} - \sigma_g$. In contrast suppose the government plays $a_g \geq y$ and take the limiting case that playing $a_g = y$ fully avoids shame (limiting case as $\beta \rightarrow \infty$). The payoff from this effort is less than or equal to $y - \frac{y^2\lambda_g}{2}$. Comparing these payoffs, the former is larger if $y \geq \frac{1+\sqrt{2\lambda_g\sigma_g}}{\lambda_g}$. Hence when this condition holds, the government prefers to play some $a_g \in [1/\lambda_g, y)$, than any $a_g \geq y$.

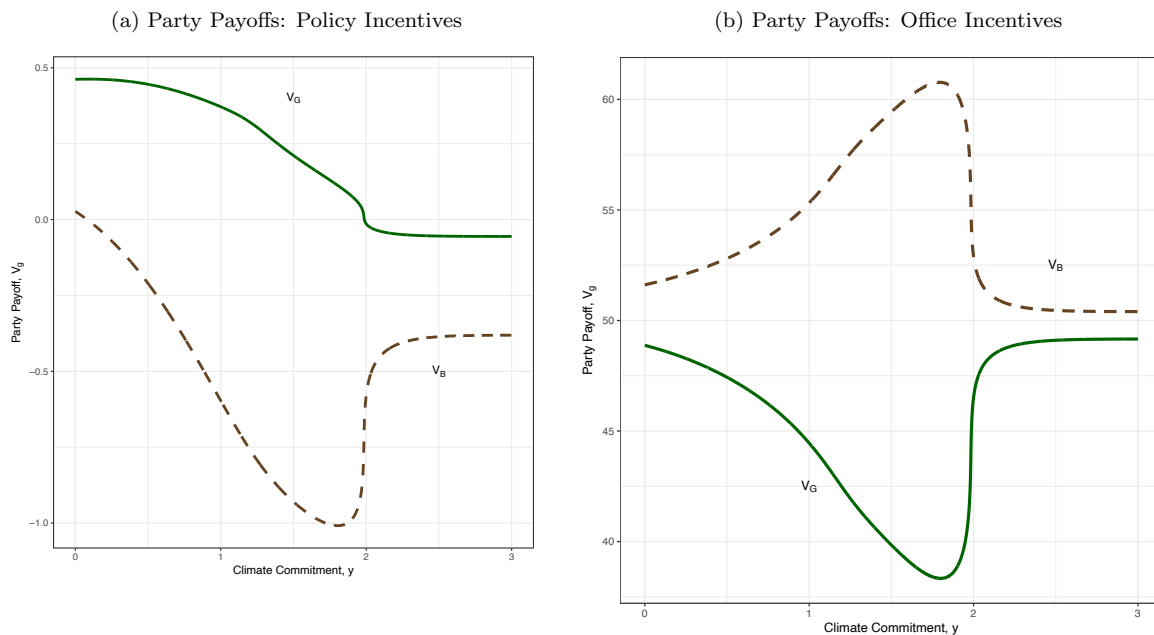
For part 3, when $foc = 0$ holds then, $\lambda_g a_g^* - 1 = \sigma_g \sqrt{\beta} \phi(\sqrt{\beta}(y - a_g^*))$. Substitute the RHS into SOC: $soc = -\lambda_g + \beta(y - a_g^*)(\lambda_g a_g^* - 1)$. Since the $u_g(\cdot)$ is continuous in a_g , there can only be two local maxima if there is also a local minimum between them. The soc expression is maximized by $a_g = \frac{y+1/\lambda_g}{2}$ which yields a maximum of $-\lambda_g + \frac{\beta + \beta\lambda_g^2 y^2 - 2\beta\lambda_g y}{4\lambda_g}$. Hence provided that $\beta < \frac{4\lambda_g^2}{(\lambda_g y - 1)^2}$, the soc expression is negative for all $foc = 0$ and so there cannot be a local minimum. Absent a local min there must be a unique maximum. In contrast if signals are relatively precise, $\beta > \frac{4\lambda_g^2}{(\lambda_g y - 1)^2}$, then there can be two local maxima that satisfy $foc = 0$ and the best effort a_g^* can be discontinuous in y .

Part 4 is simply the limiting case elaborated on in the text. If $a_g < y$, then $u_g(a_g) = A - \frac{\lambda_g a_g^2}{2} - \sigma_g$ which is maximized by $a_g = \frac{1}{\lambda_g}$. If $a_g > y$, then $u_g(a_g) = A - \frac{\lambda_g y^2}{2}$, which for $y > \frac{1}{\lambda_g}$ is maximized by $a_g = y$. The condition $y = \frac{\sqrt{2}\sqrt{\lambda_g}\sqrt{\sigma_g}+1}{\lambda_g}$ follows directly from equating these payoffs. ■

Party Payoffs in the General Model

We plot parties' payoffs V_B and V_G in Figure A.1. The left panel of the figure shows party payoffs as a function of y if parties only care about policy outcomes. By setting y , a party can influence the policy choice of the other party. For instance, G can tie B 's hands in terms of enacting greater mitigation efforts after the election. As y becomes more ambitious, B 's payoff decreases quite substantially as it exerts effort further and further from its ideal point to meet the pledge. By contrast, since G would be willing to implement more ambitious mitigation strategies *ex ante*, its payoff decreases less dramatically as it incurs the costs of exerting effort to meet an increasingly ambitious commitment. For sufficiently high y , it becomes too costly for either party to meet the commitment, and they revert to implementing their ideal points, knowing that it is likely that they will be shamed. In this case, parties generically prefer a lower commitment so it will be easy for them to both implement their ideal point and avoid shaming.

Figure A.1: Party Payoffs as a Function of Commitments



In the right panel of Figure A.1, we plot party payoffs if their main incentive in pursuing climate commitments is to remain in office. Parties' considerations change dramatically when they select commitments in order to maximize electoral success. As we describe in the limiting cases, despite their *ex ante* distaste for climate action, the Brown party may have incentives to set a climate commitment that is highly ambitious. In so doing, B can set a target that is too high for them to meet, knowing they will likely be

shamed if they win the election, but G will attempt to pursue it. G 's adventurous mitigation efforts then appear extremely costly for the voter, who knows that B , in failing to meet the commitment, will exert effort closer to the voter's ideal point. Office-holding concerns can therefore generate counterintuitive cases in which anti-climate governments set more ambitious climate commitments than pro-climate governments, knowing full well that they will not be honored, but are made in order to leverage the fact that pro-climate governments would become less electorally attractive to voters.

Generalization and Proofs of Winning Office Limiting Case

For the precise shaming limiting case ($\beta \rightarrow \infty$), it is useful to restate some definitions and define several new quantities:

1. $\hat{y}_g = \frac{1 + \sqrt{2\sigma_g \lambda_g}}{\lambda_g}$ is the highest commitment g will implement before preferring to implement its ideal point and be shamed.
2. $\hat{\sigma} = \frac{(\lambda_B - \lambda_G)^2}{2\lambda_G^2 \lambda_B}$ is the minimum shaming cost such that B prefers to implement G 's ideal point rather than implement its own ideal point and be shamed: $u_B(\tilde{a}_G) = u_B(\tilde{a}_B) - \hat{\sigma}$.
3. $\bar{y} = \frac{2\lambda_B - \lambda_M}{\lambda_B \lambda_M}$ is the policy commitment (above \tilde{a}_B) such that, if implemented, the median voter would be indifferent between \bar{y} and B 's ideal point.
4. $\bar{\sigma} = \frac{2(\lambda_M - \lambda_B)^2}{\lambda_B \lambda_M^2}$ is the smallest shaming cost such that B would implement \bar{y} if elected (i.e. $u_B(\bar{y}) = u_B(\tilde{a}_B) - \bar{\sigma}$).
5. $\bar{\bar{\sigma}} = \frac{(\lambda_M - \lambda_B)^2}{2\lambda_M^2 \lambda_B}$ is that smallest shaming cost such that B can implement the median voter's ideal point: $u_B(\tilde{a}_M) = u_B(\tilde{a}_B) - \bar{\bar{\sigma}}$.
6. $\hat{\hat{\sigma}}$ is defined such that $\Delta(\tilde{a}_G, \tilde{a}_M; y = \tilde{a}_M) = \Delta(\hat{y}_G, \tilde{a}_B; y = \hat{y}_G)$. This is the smallest shaming cost such that the largest commitment that G can credibly implement produces the same electoral bias as B committing to the median voter's ideal point ($y = \tilde{a}_M$).

To limit the analysis to substantively interesting cases, we make the following assumption:

Assumption 1 *If $\sigma_B > \hat{\sigma}$ then $\sigma_G > \frac{(\lambda_B - \lambda_G)^2 - 2\sqrt{2}\sqrt{\lambda_B \lambda_G}(\lambda_B - \lambda_G)\sqrt{\sigma_B}}{2\lambda_B^2 \lambda_G} + \frac{\lambda_G \sigma_B}{\lambda_B}$.*

This condition ensures that when B can commit to a policy above \tilde{a}_G that $\hat{y}_G > \hat{y}_B$, which substantively means that the Green party can implement larger commitments than the Brown party. The condition is only violated if Brown's shaming cost vastly exceeds Green's, such that Brown can commit to providing more policy than Green. Such a case seems substantively unlikely.

The proposition below specifies the optimal commitments for office-seeking parties.

Proposition A.1 *Let $\beta \rightarrow \infty$, $\Psi \rightarrow \infty$, $F' > 0$ and Assumption 1 holds. G 's optimal climate commitment*

is

$$y_G^* = \begin{cases} y \leq \tilde{a}_B \text{ or } y \in (\hat{y}_B, \tilde{a}_G] & \text{if } \sigma_B < \bar{\sigma} \\ \hat{y}_B = \frac{1 + \sqrt{2\sigma_B \lambda_B}}{\lambda_B} & \text{if } \bar{\sigma} \leq \sigma_B \leq \hat{\sigma} \\ \tilde{a}_G & \text{if } \sigma_B > \hat{\sigma}. \end{cases}$$

If $\sigma_B \geq \bar{\sigma}$ then B 's optimal commitment is

$$y_B^* = \begin{cases} \tilde{a}_M & \text{if } \sigma_G \leq \hat{\sigma} \\ \hat{y}_G = \frac{1+\sqrt{2\sigma_G\lambda_G}}{\lambda_G} & \text{if } \sigma_G > \hat{\sigma}. \end{cases}$$

If $\sigma_B < \bar{\sigma}$ then B 's optimal commitment is

$$y_B^* = \begin{cases} \hat{y}_B = \frac{1+\sqrt{2\sigma_B\lambda_B}}{\lambda_B} & \text{if } \Delta(\tilde{a}_G, \hat{y}_B; \hat{y}_B) \geq \Delta(\hat{y}_G, \tilde{a}_B; \hat{y}_G) \\ \hat{y}_G = \frac{1+\sqrt{2\sigma_G\lambda_G}}{\lambda_G} & \text{if } \Delta(\tilde{a}_G, \hat{y}_B; \hat{y}_B) < \Delta(\hat{y}_G, \tilde{a}_B; \hat{y}_G) \end{cases}$$

Proof of Proposition A.1: Since officeholding dominates, Green seeks to maximize $\Delta(a_G^*, a_B^*; y)$; while Brown seeks to minimize $\Delta(a_G^*, a_B^*; y)$. We consider each case.

First suppose that G is the incumbent and B 's shaming cost is small: $\sigma_B < \bar{\sigma}$. All the equilibrium commitments ($y \leq \tilde{a}_B$ or $y \in (\hat{y}_B, \tilde{a}_G]$) result in B and G each implementing their ideal point. Can G do better? No, if G 's commitment is above \tilde{a}_G then either G implements a policy above its ideal point (which is bad both in terms of policy and electability) or G is shamed. So G never profits by $y > \tilde{a}_G$. If G 's commitment is $y \in (\tilde{a}_B, \hat{y}_B]$ then if elected B would implement this commitment, which is closer to the median voter's ideal point than \tilde{a}_B ; this would reduce the electoral bias and harm G 's electoral prospects.

Second, consider the case of a moderate shaming cost: $\bar{\sigma} \leq \sigma_B \leq \hat{\sigma}$. The largest commitment that B would implement is $\hat{y}_B = \frac{1+\sqrt{2\sigma_B\lambda_B}}{\lambda_B}$, which is above the median voter's ideal point and above \bar{y} . In this range, the electoral bias is increasing in y , subject to y being implemented by B . Hence G maximizes electoral bias by a commitment to the maximizes the policy that B implements.

Finally if $\sigma_B > \hat{\sigma}$, then any commitment $y \in [\tilde{a}_G, \hat{y}_B]$ results in both parties implementing the same post election policy, which maximizes the electoral bias. Within this set of electorally optimal policies, G prefers that its ideal point is implemented. Hence $y_G^* = \tilde{a}_G$.

Now consider B 's optimal commitments. The analysis is split into two cases. First suppose that B 's shaming cost is sufficiently large that B can implement the median voter's ideal point: $\sigma_B \geq \bar{\sigma}$. As we saw from the discussion of Figure 3, for all $y \leq \tilde{a}_G$, the median voter's ideal point minimizes $\Delta(a_G^*, a_B^*; y)$. If B proposes $y > \tilde{a}_G$, then $\Delta(a_G^*, a_B^*; y)$ is minimized by pledging \hat{y}_G , the largest policy that G will implement. Note that by assumption 1, at this pledge, B would renege and be shamed. Thus, B 's optimal choice will be a policy that minimizes one of the two following electoral biases, $\Delta(\hat{y}_G, \tilde{a}_B^*; \hat{y}_G)$ or $\Delta(\tilde{a}_G, \tilde{a}_M; y = \tilde{a}_M)$. When G 's shaming cost is large ($\sigma_G > \hat{\sigma}$), then the former is the optimal as it produces the greatest electoral

bias in B 's favor; and when shaming cost is smaller then the latter is optimal.

Second, suppose B 's shaming cost is insufficient for B to implement the median voter's ideal point: $\sigma_B < \bar{\sigma}$. The analysis is similar to that case above, however, now B cannot commit to the median voter's ideal policy. Instead B picks between the largest policy that it can implement (\hat{y}_B) or the largest policy that G can implement. The electoral biases for these pledges are $\Delta(\tilde{a}_G, \hat{y}_B; \hat{y}_B)$ and $\Delta(\hat{y}_G, \tilde{a}_B; \hat{y}_G)$, respectively. Given the primacy of office holding, B selects the pledge with the largest electoral bias in B 's favor. ■

In Proposition 3, G has a range of optimal commitments when $\sigma_B < \bar{\sigma}$; however, all such commitments result in an observationally equivalent outcome where the commitment does not affect B 's downstream effort to implement policy at its ideal point. To plot Figure 4, we use the equilibrium refinement that selects the largest commitment that G would implement (that results in no shaming for B).